Model-Independent Dark Matter Properties from Cosmic Growth

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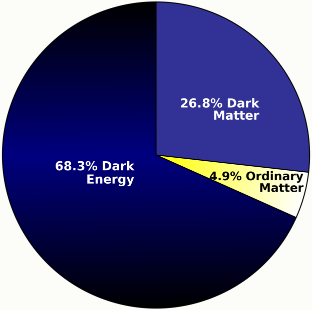
Dark Matter in Modern Cosmology



- Dark matter (DM) constitutes $\sim 6\times$ the baryonic matter abundance.
- Vital for structure formation, galaxy rotation curves, gravitational lensing, and cosmic microwave background (CMB) power spectrum.
- Only interacts gravitationally (so far as known); composition and deeper properties remain unclear.

Fundamental Inquiry

Key question:

Is DM exactly cold, pressureless, and gravitationally "vanilla"?

More questions:

- Warm or interacting DM?
- Multiple DM species with distinct behaviors?
- Conversion between dark matter and dark energy?
- Environmental dependence (screening or modified gravity)?
- *Time-varying clustering strength?*
- *Non-zero equation of state $w_{\rm dm}$?*

Dark Matter and Cosmic Growth

- In linear, subhorizon regime, growth of matter overdensity $\delta(t) = \delta\rho_m/\rho_m$ is governed by:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N(\rho_b + \rho_{\rm dm})\delta = 0 \tag{1}$$

- Standard model:

$$G_N \text{ (same gravity as baryons)}, \quad w_{\rm dm} = 0 \quad \text{(pressureless)}.$$

Growth of Structure as a Probe

- Cosmic structure formation probes such as **Euclid**, **DESI**, and **Peculiar Velocity** tell more than just the expansion rate of the universe.
- It is sensitive to:
  - The clustering strength, whether gravitational or due to other interactions.
  - The energy density evolution (equation of state) of each component.

Model-Independent Modifications

Things modified:

1. Modified Clustering Strength:

$$G_N \rho_m \delta \quad \rightarrow \quad G_N \left(\rho_b \delta_b + F_{\rm cl}\, \rho_{\rm dm} \delta_{\rm dm}\right)$$

2. Modified Dark Matter Density Evolution:

$$\rho_{\rm dm}(a) \quad \rightarrow \quad \rho_{\rm dm}(a=1)\, a^{-3}\, F_{\rm eos}(a)$$

Things preserved:

- Dark energy sector: cosmological constant.
- Baryon density evolution: $\rho_b \sim a^{-3}$:

$$\Omega_{b,0} h^2 = 0.02233$$

- Hubble parameter: $h = 0.7 \quad \Rightarrow \quad \Omega_b = 0.0456$
- Fiducial matter density:

$$\Omega_{m,0}^{\Lambda\rm CDM} = 0.3$$

Model-independent approach via redshift binning

Cosmic history divided into 4 redshift bins:

$$z \in [4,\, 2], \quad [2,\, 1], \quad [1,\, 0.5], \quad [0.5,\, 0]$$

In each bin, we allow:

1. Deviation in clustering strength:

$$F_{\mathrm{cl}}(z) = 1 + c_i$$

2. Deviation in the equation of state:

$$w_{\mathrm{dm}}(z) = w_i$$

For $z > 4$, we assume standard ΛCDM:

$$F_{\mathrm{cl}} = 1, \quad w_{\mathrm{dm}} = 0$$

Constraining the deviations

Forecast:

- Mock data assumes:
  - **euclid**: $z = 1.65$ to 1.95 (early)
  - **desi**: $z = 0.35$ to 1.55 (mid)
  - **pv**: $z = 0.1$ (late)

  in steps of 0.1.

Fisher Matrix:

- For independent measurements of the observables:

$$F_{ij} = \sum_z \frac{\partial O(z)}{\partial p_i} \frac{1}{\sigma^2(z)} \frac{\partial O(z)}{\partial p_j}.$$

  where $O(z)$ is an observable and $p_i = [c_i, w_i]$
- $F^{-1} =$ **parameter covariance matrix**.

Implementing the Fisher Approach

Step-by-step:

1. **Choose observables:** $f\sigma_8(z_i)$ not $g(z_i)$
2. **Specify errors/covariance:** 2%
3. **Compute sensitivities:** $\partial\mathcal{O}/\partial p_i$ at the fiducial model.
4. **Assemble Fisher matrix:** $F_{ij} = \sum_z \ldots$
5. **Invert** to obtain F^{-1} and interpret $\sigma(p_i)$ & parameter correlations.

Growth equation in terms of $g(a)$

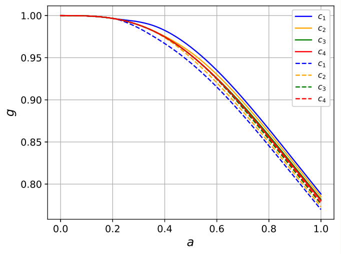
$$a^2 \frac{d^2 g}{da^2} + \left(5 + \frac{1}{2}\frac{d\ln H^2}{d\ln a}\right) a \frac{dg}{da} + \left(3 + \frac{1}{2}\frac{d\ln H^2}{d\ln a} - \frac{3}{2}\left[\Omega_b(a) + F_{\rm cl}(a)\,\Omega_{\rm dm}(a)\right]\right) g = 0 \qquad (2)$$

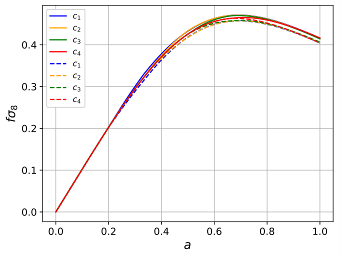
Key quantities:

- $g(a) = \dfrac{\delta(a)/\delta(a_i)}{a/a_i}$ (normalized growth relative to matter domination)
- $f = \dfrac{d\ln \delta}{d\ln a} = 1 + \dfrac{d\ln g}{d\ln a}$
- Observable: $f\sigma_8(a) = \dfrac{\sigma_8^{\Lambda\rm CDM}}{g_0^{\Lambda\rm CDM}}\, ag\left(1 + \dfrac{d\ln g}{d\ln a}\right)$

A) Clustering strength. Effects of $c_i = \pm 0.05$ and $w_i = 0$

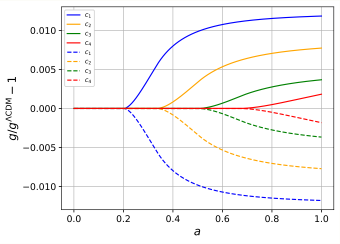
- **Key effect:** Growth is dynamical — early deviations (c_1) influence $g(a)$ and $f\sigma_8(a)$ over extended time.
- Late-time deviations (c_4) have more localized impact.

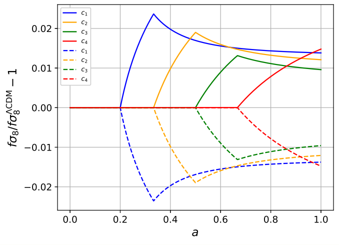




Residual relative to ΛCDM

- Early deviations (e.g. c_1) have a **long-lasting impact** on $g(a)$, due to the integrated nature of growth.
- The response in $f\sigma_8(a)$ is more **localized**, showing sharp changes near the bin edges.
- These residuals highlight the **dynamical memory** of structure growth to past deviations.



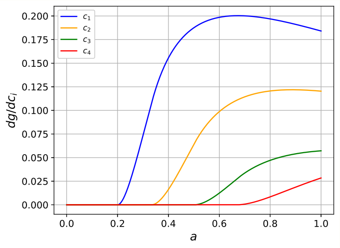


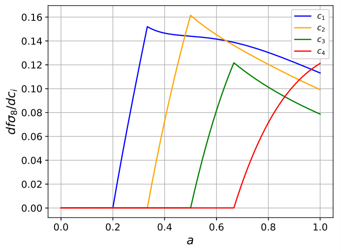
Sensitivity Curves

- Sensitivity curves show $\partial\mathcal{O}/\partial c_i$ for:

$$\mathcal{O} = g(a) \quad \text{(left)}, \quad f\sigma_8(a) \quad \text{(right)}$$

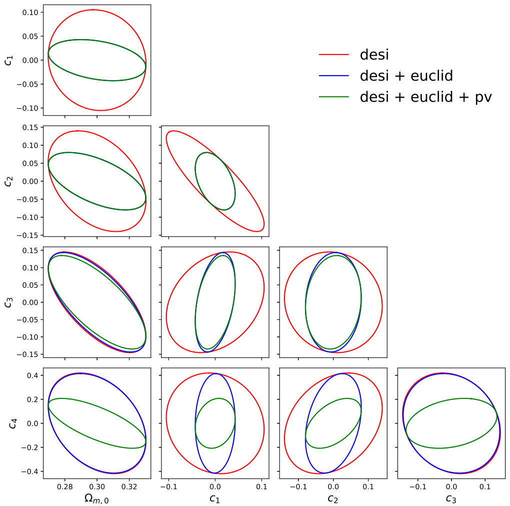
- Different shapes for each c_i enable tomographic data to **break degeneracies** between bins.
- Sensitivity in $f\sigma_8$ rises **approximately linearly** at high z, as expected from its integral dependence on clustering history.





Constraints on Clustering Deviations

Estimated constraints (all data): $\sigma(c_i) = (0.0282,\ 0.0524,\ 0.0888,\ 0.1367)$
Strongest correlation: $r(c_2, c_4) = 0.49$



B) Equation of State: Modified Evolution

$$\rho_{\rm dm}(a) = \rho_{\rm dm}(a=1)\, e^{3\int_a^1 \frac{da'}{a'}[1+w(a')]}$$

Piecewise implementation in bins:

$$\rho_{\rm dm}(a) = \rho_{\rm dm}^{\Lambda\rm CDM}(a=1)\, a^{-3} \prod_{i=1}^{j} \left(\frac{\min[a, a_{i+1}]}{a_i}\right)^{-3w_i} \tag{7}$$

Effective present-day DM density:

$$\Omega_{\rm dm,0} = \Omega_{\rm dm,0}^{\Lambda\rm CDM} \cdot a_1^{3w_1} a_2^{3(w_2-w_1)} a_3^{3(w_3-w_2)} a_4^{3(w_4-w_3)} \tag{8}$$

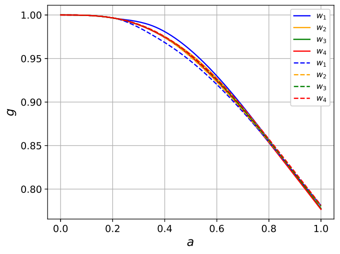
At high redshift:

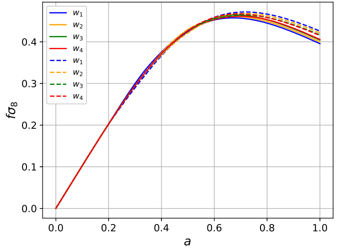
$$H^2(a<a_1)/H_0^2 = \Omega_{m,0}^{\Lambda\rm CDM} a^{-3} + 1 - \Omega_{b,0} - \Omega_{\rm dm,0} \tag{9}$$

Implication: Even if $w_i \neq 0$ only in a bin, background evolution ($\Omega_{\rm dm,0}$, H^2) is globally affected.

Effects of $w_i = \pm 0.05$ and $c_i = 0$

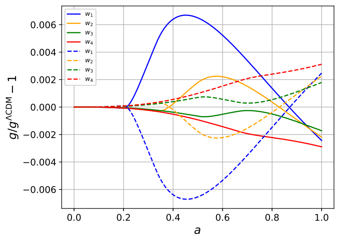
Impact of individual $w_i = \pm 0.05$ on $g(a)$ and $f\sigma_8(a)$.

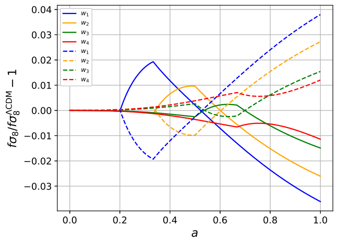




Residuals from ΛCDM

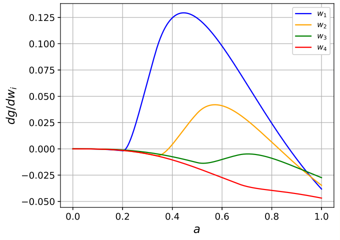
- Three phases in the residual behavior:
  1. **Pre-bin offset:** due to modified $\Omega_{m,0}$ even when $w_i \neq 0$ only inside the bin.
  2. **In-bin response:** direct effect on Hubble friction and source term alters growth rate.
  3. **Post-bin memory:** growth retains a weaker residual due to changed background.
- Deviations in w_i induce both direct (within-bin) and indirect (background-altering) effects.

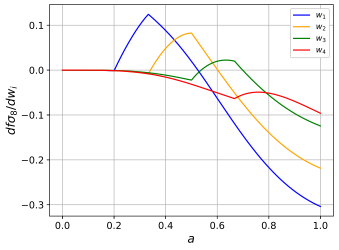




Sensitivity to Equation of State Deviations

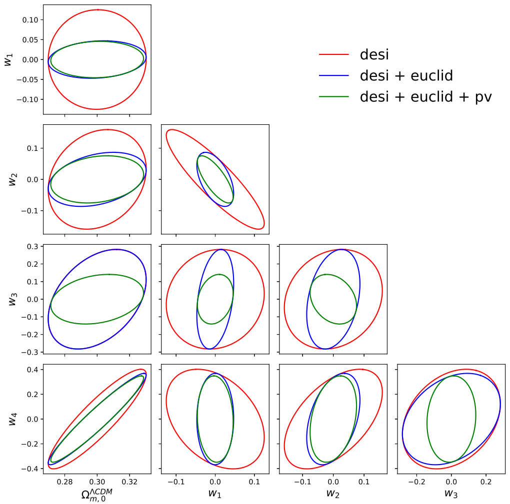
Overall sensitivity in $f\sigma_8$ is **weaker** and more degenerate than for clustering deviations.





Constraints on Equation of State Deviations

Final constraints (all data): $\sigma(w_i) = (0.0300,\ 0.0497,\ 0.0926,\ 0.2293)$
Strongest correlation: $r(w_1, w_2) = -0.75$



Interpreting EoS Constraints

What does the figure tell us?

- The contours shrink with more data:
  - **DESI**-only: weakest constraints.
  - Adding **Euclid**: significantly improves w_1, w_2 due to better high-z coverage.
  - Adding **PV**: minor improvement, mostly for w_3; little gain on w_4 due to degeneracy.
- The strongest correlation appears between w_1 and w_2 — typical of adjacent bin parameters.
- Degeneracy with $\Omega_{m,0}$ is strongest for w_4, limiting low-z precision.
- All parameters are constrained independently: low correlation coefficients and no extreme degeneracies.

Bottom line: Model-independent constraints on $w_{\rm dm}(z)$ are achievable — though limited by degeneracies and redshift coverage.

Conclusions

- We investigated whether dark matter is **strictly cold, pressureless, and gravitationally standard**.
- **Model-independent approach** using redshift-binned deviations in:
  1. Clustering strength $F_{\rm cl}(z)$
  2. Equation of state $w_{\rm dm}(z)$
- Used **mock** $f\sigma_8$ **data** from DESI-, Euclid-, and PV-like surveys, with Fisher matrix formalism.
  1. Clustering deviations constrained to:

$$\sigma(c_i) = (0.028,\ 0.052,\ 0.089,\ 0.137)$$

  2. EoS deviations constrained to:

$$\sigma(w_i) = (0.030,\ 0.050,\ 0.093,\ 0.229)$$

- Sensitivity analysis revealed:
  - Growth retains memory of early deviations ("inertia" in $g(a)$)
  - Wide redshift coverage is key to breaking degeneracies
- **Outlook:** Future high-redshift surveys can further tighten these model-independent tests of dark matter properties.

Thank You!

Questions?