기초과학연구원
Institute for Basic Science

A Match Made in Heaven:

Linking Observables in Inflationary Cosmology

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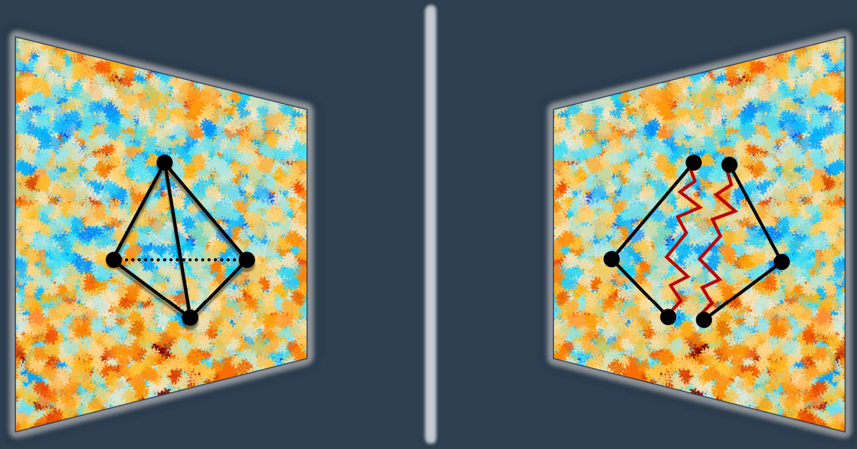
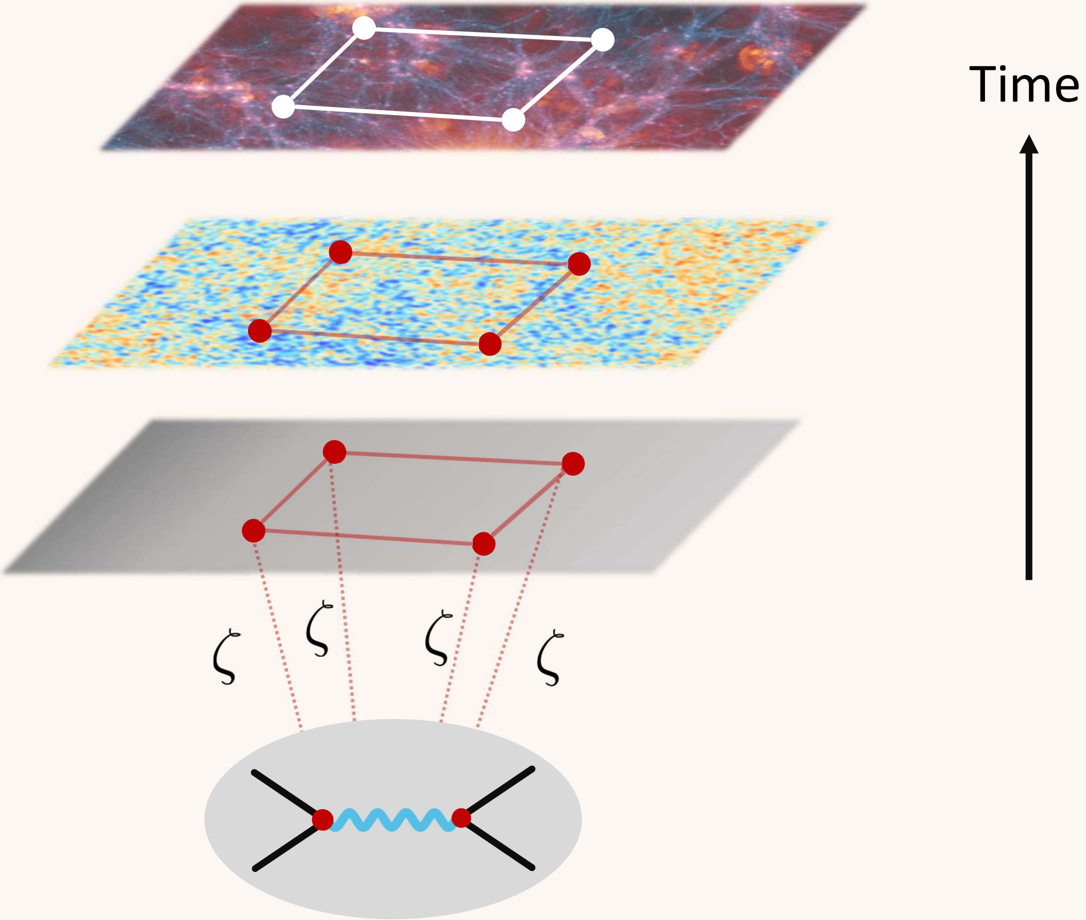
@Prague| 07 April 2025



In collaboration with: David Stefanyszyn and Xi Tong

Based on:
2504.XXXX
2406.00099
2309.07769

Cosmological correlators



LSS $\langle \delta_m^n \rangle$

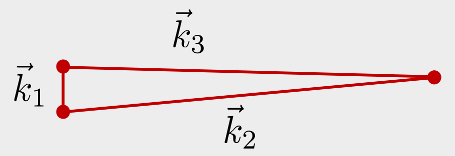
CMB $\langle \delta T^n \rangle$

End of inflation $\langle \zeta^n \rangle$

Time

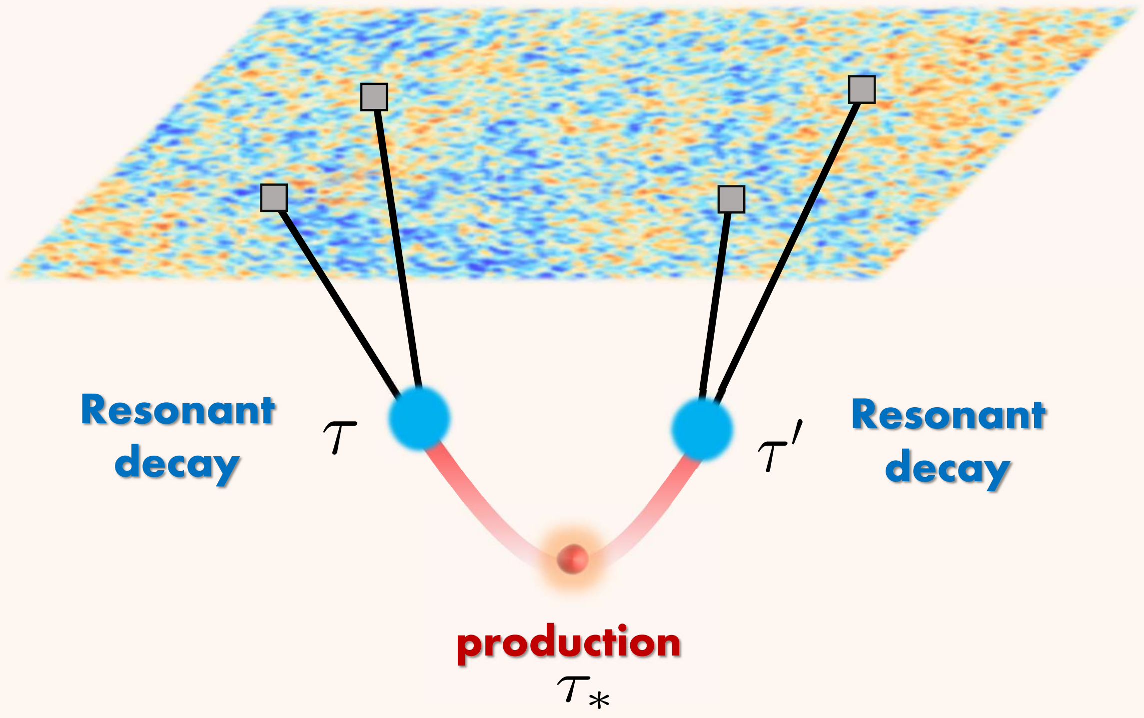
Inflation as the Cosmological Collider

Inflation as the Cosmological Collider (CC)



Soft limit

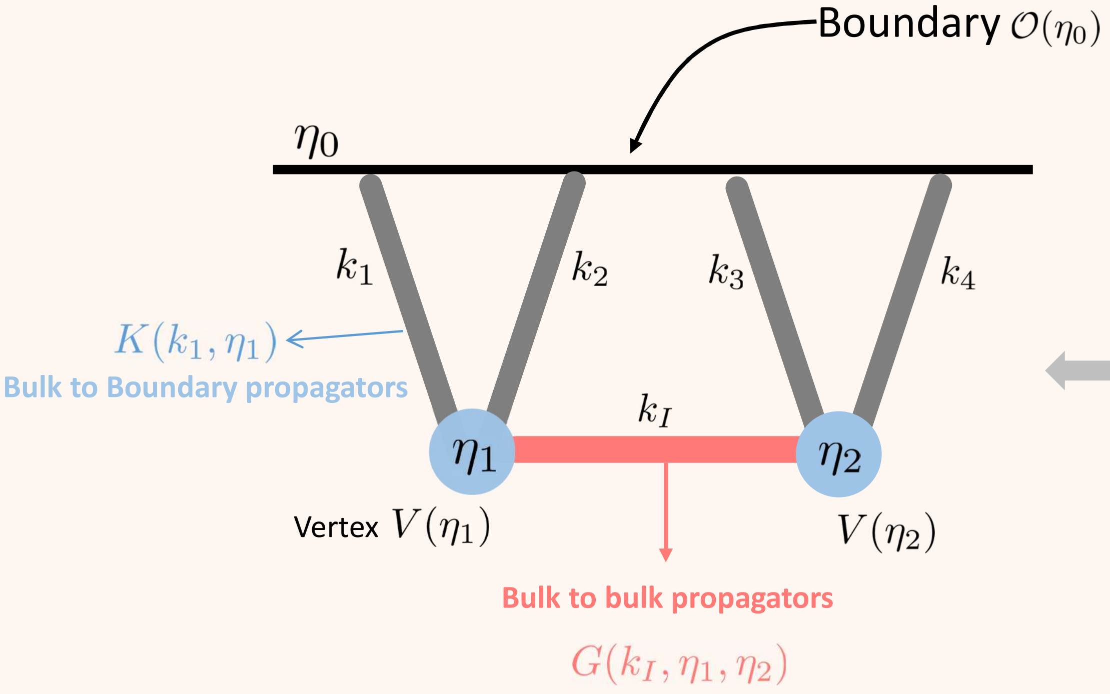
- Inflationary particle spectrum



$$S \sim \mathcal{A}(\lambda, m)\, e^{-\pi\mu} \left(\frac{k_3}{k_1}\right)^{1/2} \sin\left[\underset{\substack{\Downarrow \\ \text{mass}}}{\mu} \log\left(\frac{k_3}{k_1}\right) + \vartheta\right] \underset{\substack{\Downarrow \\ \text{spin}}}{P_s}(\cos\theta)$$

$$\mu = \begin{cases} \sqrt{\dfrac{m^2}{H^2} - \dfrac{9}{4}} & s = 0 \\ \sqrt{\dfrac{m^2}{H^2} - \left(s - \dfrac{1}{2}\right)^2} & s \neq 0 \end{cases}$$

Analytical Calculation of cosmological correlators



Boundary $\mathcal{O}(\eta_0)$

$K(k_1,\eta_1)$

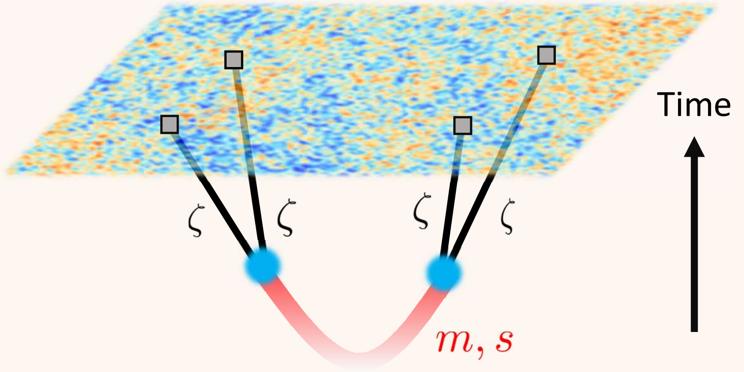
Bulk to Boundary propagators

Vertex $V(\eta_1)$

$V(\eta_2)$

Bulk to bulk propagators

$G(k_I,\eta_1,\eta_2)$



$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle' = \int d\eta_1 \int d\eta_2 V(\eta_1)V(\eta_2)\boxed{G(k_I,\eta_1,\eta_2)}K(k_1,\eta_1)K(k_2,\eta_1)K(k_3,\eta_2)K(k_4,\eta_2)$$

Too difficult to solve analytically!

(1) Nested Time integral

$$G \sim \theta(\eta_1-\eta_2)v_k(\eta_1)v_k^*(\eta_2)+\theta(\eta_2-\eta_1)v_k^*(\eta_1)v_k(\eta_2)$$

(2) Mode functions are complicate

$$v_k \sim H_{i\mu} \text{ or } W_{i\kappa,i\mu}$$

BUT, the calculation of cosmological correlators is really HARD!

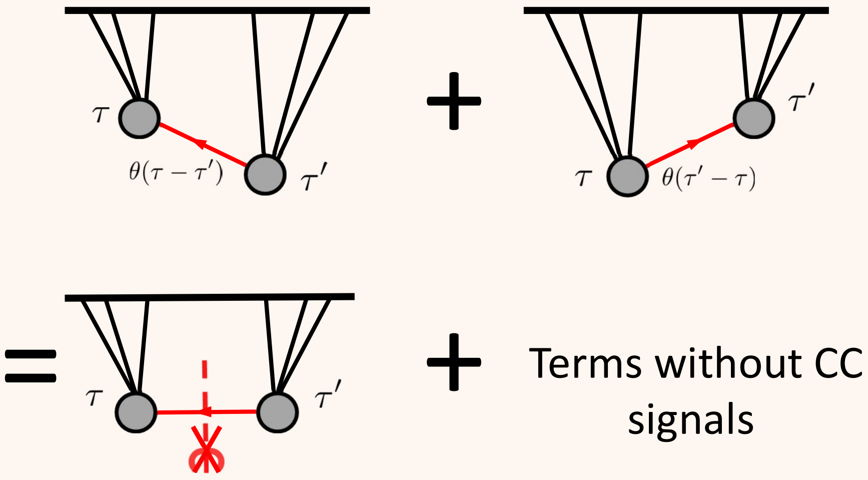
Exact	**Approximation**
Sometimes, too complicated	Sometimes, too naïve
Hard to understand all information	Hard to include all information
……	……

Trade-off

Recent Progress: **Approximation Method**

Cosmological collider cutting rule

Tong, Wang and **YHZ** [2112.03448]



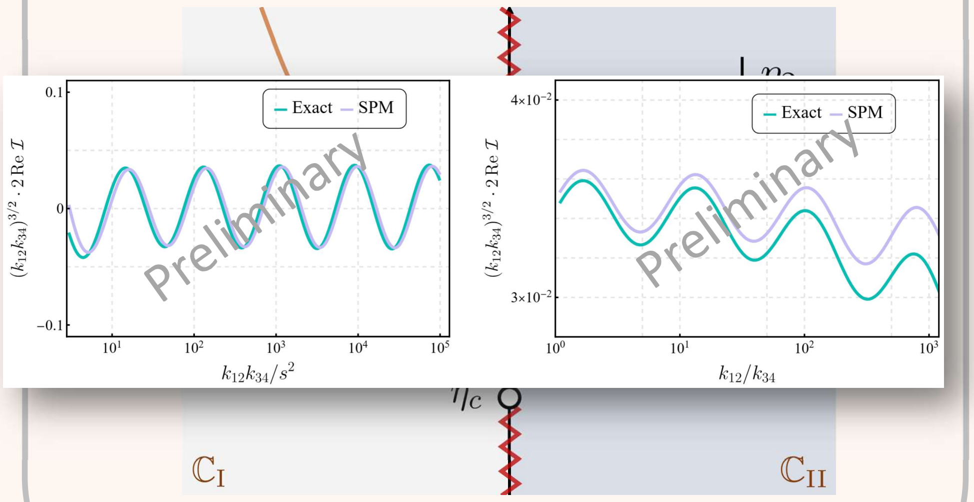
Terms without CC signals

Efficient way to extract CC signals

WKB + Saddle point method

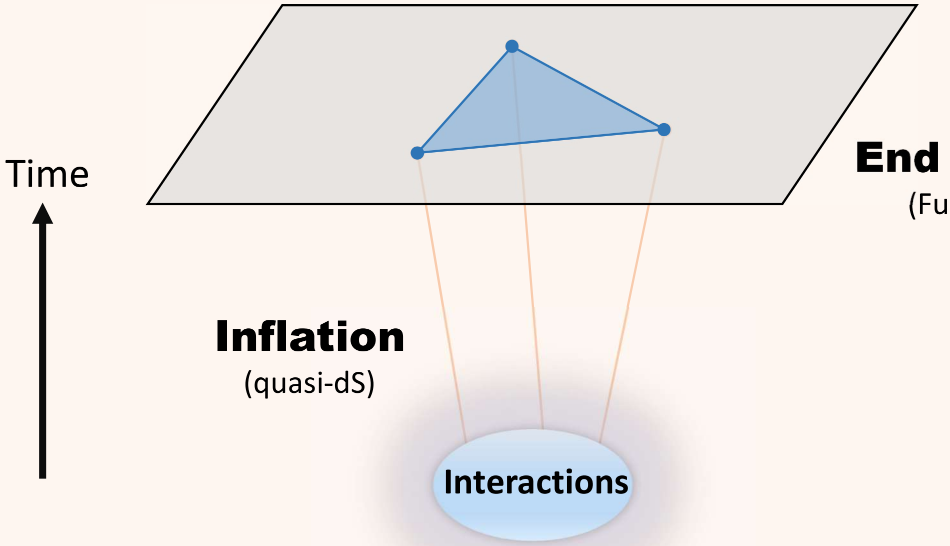
Renaux-Petel, Tong, Werth, **YHZ** [2506.XXXX]

$$\left[\frac{\partial^2}{\partial\eta^2}+w^2(s,\eta)\right]\sigma(s,\eta)=0$$



Good way to understand physics

Recent Progress: **Exact Method**



Time

Inflation

(quasi-dS)

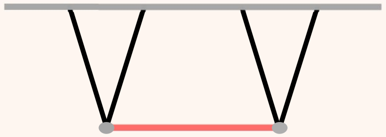
Interactions

$\eta = 0$

End of inflation

(Future boundary)

$F \sim \langle \zeta^4 \rangle$



Conformal Ward identities :

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \mu^2 + \frac{1}{4}\right] F = g^2 \frac{uv}{u+v}$$

\+ **Boundary conditions** = **fix the correlators**

dS Bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee, Pimentel 2019 2020
……

dS Symmetry : $\mathrm{d}s^2 = \dfrac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{H\eta^2}$

Translation: $P_i = \partial_i$

Rotation: $J_{ij} = x_i\partial_j - x_j\partial_i$

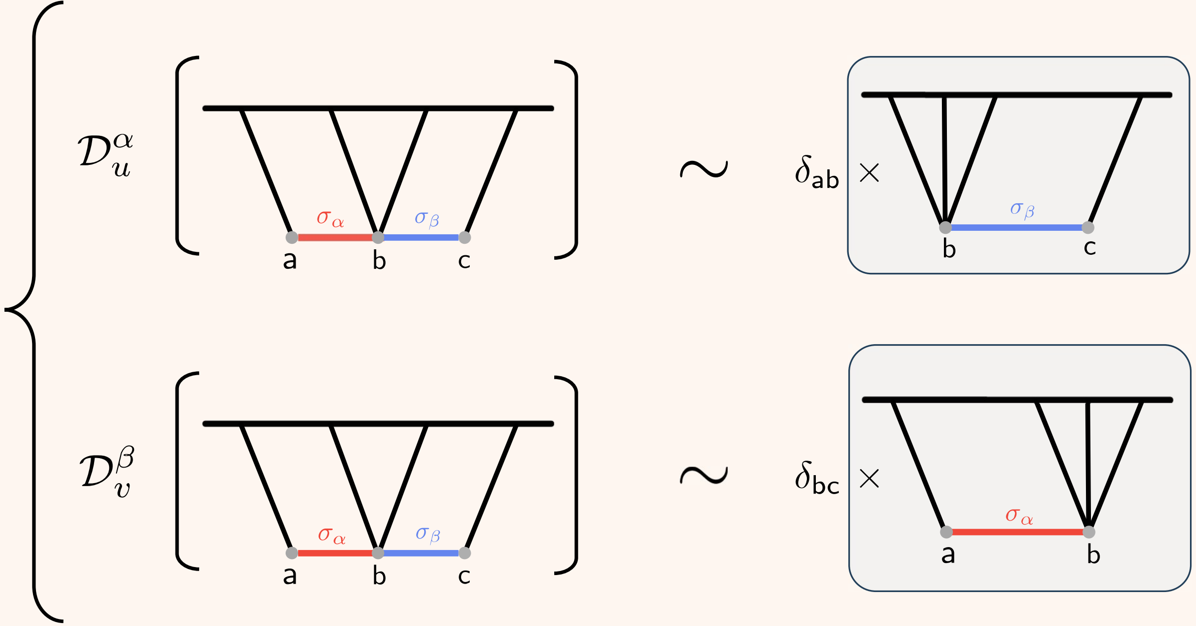
Dilation: $D = -\eta\partial_\eta - x_i\partial_i$

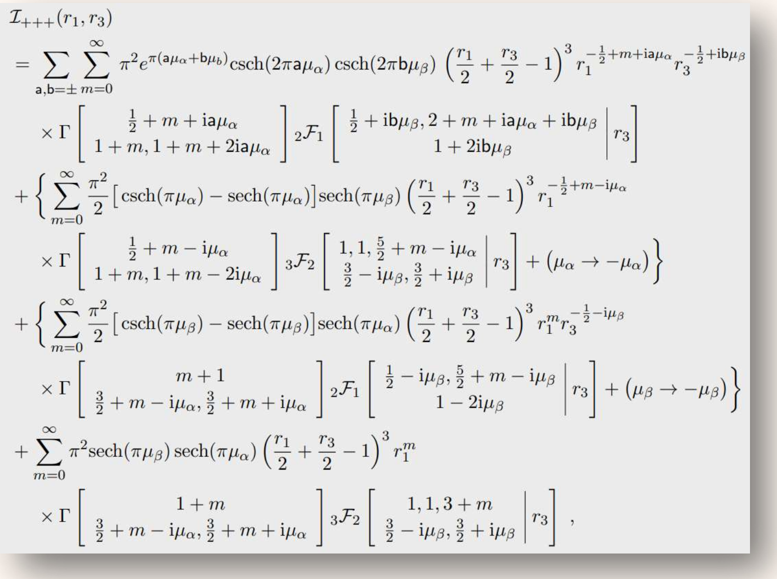
dS boosts: $K_i = 2x_i\eta\partial_\eta + \left(2x^j x_i + (\eta^2 - x^2)\delta_i^j\right)\partial_j$

Recent Progress: **Exact Method**

Double **Massive Exchange**

Aoki, Pinol, Sano, Yamaguchi and **YHZ**
[2404.09547]

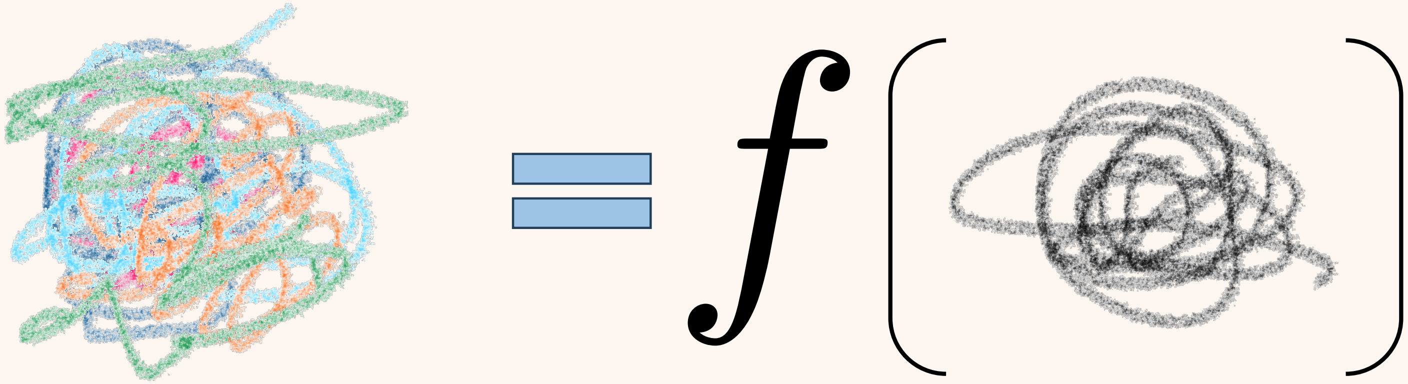




$$
\begin{aligned}
&\mathcal{I}_{+++}(r_1,r_3)\\
&=\sum_{\mathsf{a},\mathsf{b}=\pm}\sum_{m=0}^{\infty}\pi^2 e^{\pi(\mathsf{a}\mu_\alpha+\mathsf{b}\mu_b)}\operatorname{csch}(2\pi\mathsf{a}\mu_\alpha)\operatorname{csch}(2\pi\mathsf{b}\mu_\beta)\left(\frac{r_1}{2}+\frac{r_3}{2}-1\right)^3 r_1^{-\frac{1}{2}+m+\mathrm{i}\mathsf{a}\mu_\alpha} r_3^{-\frac{1}{2}+\mathrm{i}\mathsf{b}\mu_\beta}\\
&\quad\times\Gamma\left[\begin{matrix}\frac{1}{2}+m+\mathrm{i}\mathsf{a}\mu_\alpha\\ 1+m,1+m+2\mathrm{i}\mathsf{a}\mu_\alpha\end{matrix}\right]{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{1}{2}+\mathrm{i}\mathsf{b}\mu_\beta,2+m+\mathrm{i}\mathsf{a}\mu_\alpha+\mathrm{i}\mathsf{b}\mu_\beta\\ 1+2\mathrm{i}\mathsf{b}\mu_\beta\end{matrix}\right| r_3\right]\\
&+\left\{\sum_{m=0}^{\infty}\frac{\pi^2}{2}\left[\operatorname{csch}(\pi\mu_\alpha)-\operatorname{sech}(\pi\mu_\alpha)\right]\operatorname{sech}(\pi\mu_\beta)\left(\frac{r_1}{2}+\frac{r_3}{2}-1\right)^3 r_1^{-\frac{1}{2}+m-\mathrm{i}\mu_\alpha}\right.\\
&\quad\left.\times\Gamma\left[\begin{matrix}\frac{1}{2}+m-\mathrm{i}\mu_\alpha\\ 1+m,1+m-2\mathrm{i}\mu_\alpha\end{matrix}\right]{}_3\mathcal{F}_2\left[\left.\begin{matrix}1,1,\frac{5}{2}+m-\mathrm{i}\mu_\alpha\\ \frac{3}{2}-\mathrm{i}\mu_\beta,\frac{3}{2}+\mathrm{i}\mu_\beta\end{matrix}\right| r_3\right]+(\mu_\alpha\to-\mu_\alpha)\right\}\\
&+\left\{\sum_{m=0}^{\infty}\frac{\pi^2}{2}\left[\operatorname{csch}(\pi\mu_\beta)-\operatorname{sech}(\pi\mu_\beta)\right]\operatorname{sech}(\pi\mu_\alpha)\left(\frac{r_1}{2}+\frac{r_3}{2}-1\right)^3 r_1^m r_3^{-\frac{1}{2}-\mathrm{i}\mu_\beta}\right.\\
&\quad\left.\times\Gamma\left[\begin{matrix}m+1\\ \frac{3}{2}+m-\mathrm{i}\mu_\alpha,\frac{3}{2}+m+\mathrm{i}\mu_\alpha\end{matrix}\right]{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{1}{2}-\mathrm{i}\mu_\beta,\frac{5}{2}+m-\mathrm{i}\mu_\beta\\ 1-2\mathrm{i}\mu_\beta\end{matrix}\right| r_3\right]+(\mu_\beta\to-\mu_\beta)\right\}\\
&+\sum_{m=0}^{\infty}\pi^2\operatorname{sech}(\pi\mu_\beta)\operatorname{sech}(\pi\mu_\alpha)\left(\frac{r_1}{2}+\frac{r_3}{2}-1\right)^3 r_1^m\\
&\quad\times\Gamma\left[\begin{matrix}1+m\\ \frac{3}{2}+m-\mathrm{i}\mu_\alpha,\frac{3}{2}+m+\mathrm{i}\mu_\alpha\end{matrix}\right]{}_3\mathcal{F}_2\left[\left.\begin{matrix}1,1,3+m\\ \frac{3}{2}-\mathrm{i}\mu_\beta,\frac{3}{2}+\mathrm{i}\mu_\beta\end{matrix}\right| r_3\right],
\end{aligned}
$$

It becomes incredibly difficult
to make further progress…

Without knowing the analytical results,
can we still find something useful?



Maldacena’s consistency relation

$$\begin{aligned}\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle &\sim -\langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle k\frac{d}{dk}\langle\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = \\ &= -n_s\langle\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle\langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle\ , \qquad k_1 \ll k_2, k_3\end{aligned}$$

(Maldacena, 2002)

Suyama-Yamaguchi Inequality

$$\tau_{\rm NL} \geq \left(\frac{6}{5}f_{\rm NL}\right)^2,$$

(Suyama, Yamaguchi 2007)

Unitarity Cutting Rule

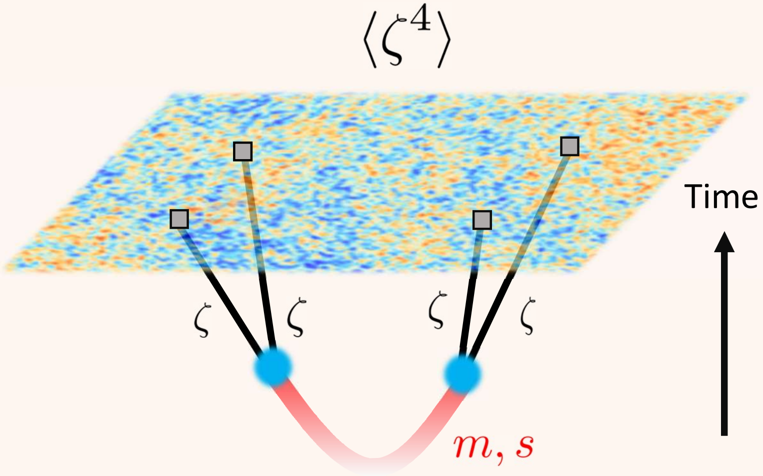
$$\begin{aligned}&\psi_4'(k_1,k_2,k_3,k_4,s,t,u) + [\psi_4'(-k_1,-k_2,-k_3,-k_4,s,t,u)]^* = \\ &P_\sigma(s)\left[\psi_3'^{\phi\phi\sigma}(k_1,k_2,s) - \psi_3'^{\phi\phi\sigma}(k_1,k_2,-s)\right]\left[\psi_3'^{\phi\phi\sigma}(k_3,k_4,s) - \psi_3'^{\phi\phi\sigma}(k_3,k_4,-s)\right]\end{aligned}$$

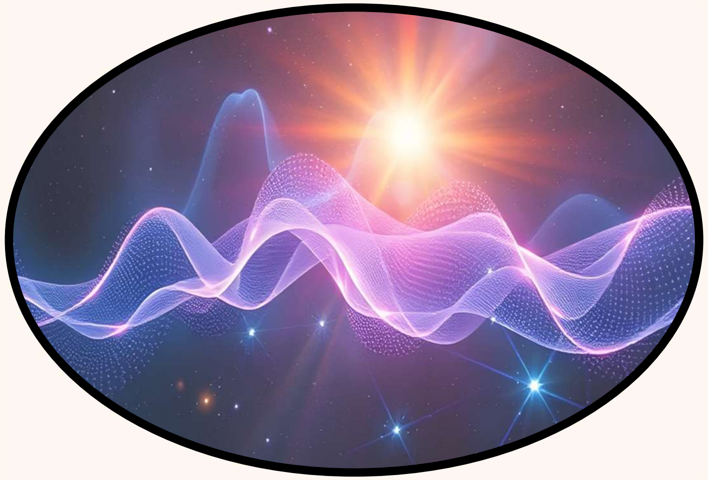
(Goodhew, Jazayeri, Pajer 2020)

Directly testable ?

(in principle)

Cosmological Correlators From Wavefunction of Universe





$$f(B_n) = 0\Psi$$

Expand the wavefunction in powers
of the field fluctuations:

Wavefunction coefficients

$$\Psi[\varphi] = \exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}\int_{\{\mathbf{k}\}} \psi_n(\mathbf{k}_1,\cdots,\mathbf{k}_n)\,\varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\,(2\pi)^3\delta^3(\mathbf{k}_1+\cdots+\mathbf{k}_n)\right]$$

Parametrization

- "Wavefunction of the universe":

$$\Psi[\varphi] = \langle\varphi|U(\eta_0,-\infty)|\mathrm{BD}\rangle = \int_{\mathrm{BD}}^{\Phi(\eta_0)=\varphi} \mathcal{D}\Phi\, e^{iS[\Phi]}$$

- Correlators from the Born rule:

$$B_n \equiv \langle\varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\rangle = \frac{\int\mathcal{D}\varphi\left|\Psi[\varphi]\right|^2\varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)}{\int\mathcal{D}\varphi\left|\Psi[\varphi]\right|^2}$$

"Cross section"

$$\rho_n(\{\mathbf{k}\})$$
$$= \psi_n(\{\mathbf{k}\}) + \psi_n^*(\{-\mathbf{k}\})$$

"Scattering Amplitude"

- **Correlators** from the Born rule:

$$B_n \equiv \langle \varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\rangle = \frac{\int \mathcal{D}\varphi \left|\Psi[\varphi]\right|^2 \varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi \left|\Psi[\varphi]\right|^2}$$

$$\Psi[\varphi] = \exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}\int_{\{\mathbf{k}\}}\psi_n(\mathbf{k}_1,\cdots,\mathbf{k}_n)\,\varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\right]$$

$$\rho_n(\{\mathbf{k}\}) = \psi_n(\{\mathbf{k}\}) + \psi_n^*(\{-\mathbf{k}\})$$

Dictionary between ρ and B

$$B_2 = -\frac{1}{\rho_2}\ ,$$

$$B_3 = -\frac{1}{{\rho_2}^3}\,\rho_3\ ,$$

$$B_4 = \frac{1}{{\rho_2}^4}\left[\rho_4 - \left(\rho_3\frac{1}{\rho_2}\rho_3 + 2\text{ perms}\right)\right]\ ,$$

$$B_5 = -\frac{1}{{\rho_2}^5}\left[\rho_5 - \left(\rho_4\frac{1}{\rho_2}\rho_3 + 9\text{ perms}\right) + \left(\rho_3\frac{1}{\rho_2}\rho_3\frac{1}{\rho_2}\rho_3 + 14\text{ perms}\right)\right]$$

General relation:

$$B_n = \frac{1}{(-\rho_2)^n}\sum_{k=0}^{n-3}(-1)^k\Big(k\text{-cuts}\Big)_\rho$$

$$\Big(k\text{-cuts}\Big)_\rho \equiv \sum_{n-k\geq n_1\cdots n_{k+1}\geq 3}\Big[\rho_{n_1}\frac{1}{\rho_2}\rho_{n_2}\cdots\rho_{n_k}\frac{1}{\rho_2}\rho_{n_{k+1}} + (\pi_{n_1\cdots n_{k+1}}-1)\text{-perms}\Big]\ .$$

Dictionary between ρ and B

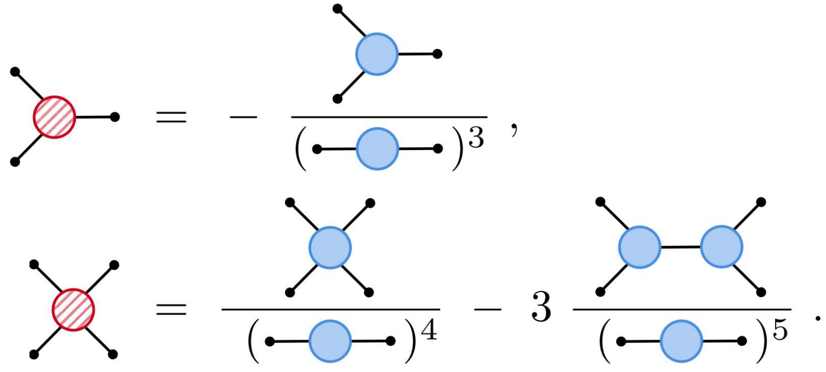
$$B_2 = -\frac{1}{\rho_2}\,,$$

$$B_3 = -\frac{1}{{\rho_2}^3}\,\rho_3\,,$$

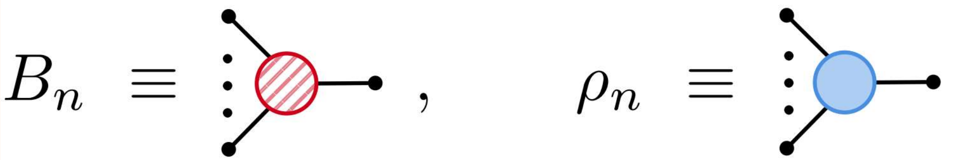
$$B_4 = \frac{1}{{\rho_2}^4}\left[\rho_4 - \left(\rho_3\frac{1}{\rho_2}\rho_3 + 2\text{ perms}\right)\right]\,,$$

$$B_5 = -\frac{1}{{\rho_2}^5}\left[\rho_5 - \left(\rho_4\frac{1}{\rho_2}\rho_3 + 9\text{ perms}\right) + \left(\rho_3\frac{1}{\rho_2}\rho_3\frac{1}{\rho_2}\rho_3 + 14\text{ perms}\right)\right]$$

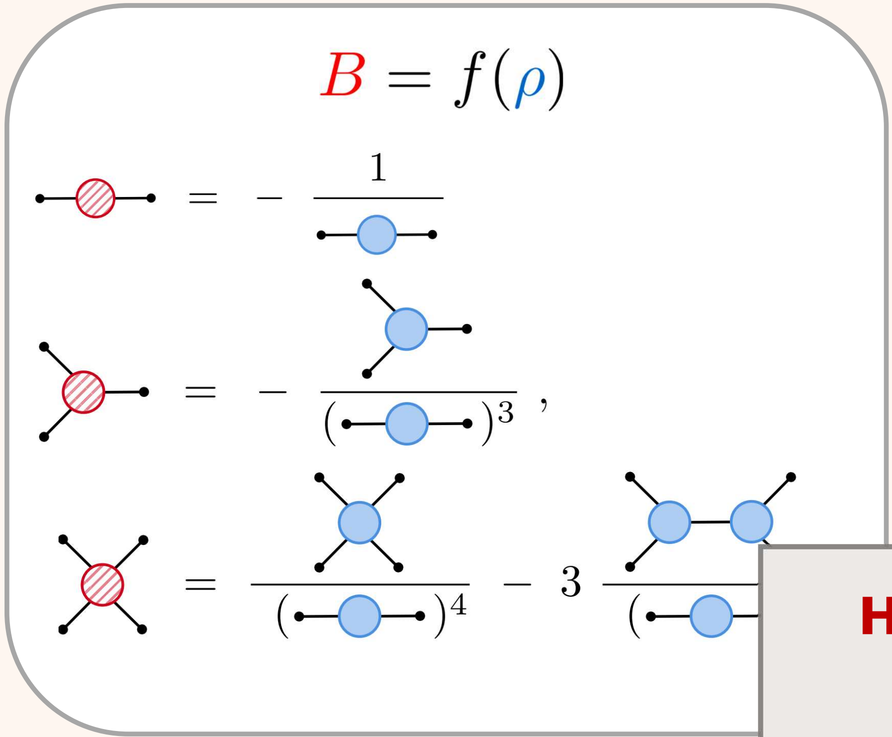
Dictionary between ρ and B



diagrammatic notations:

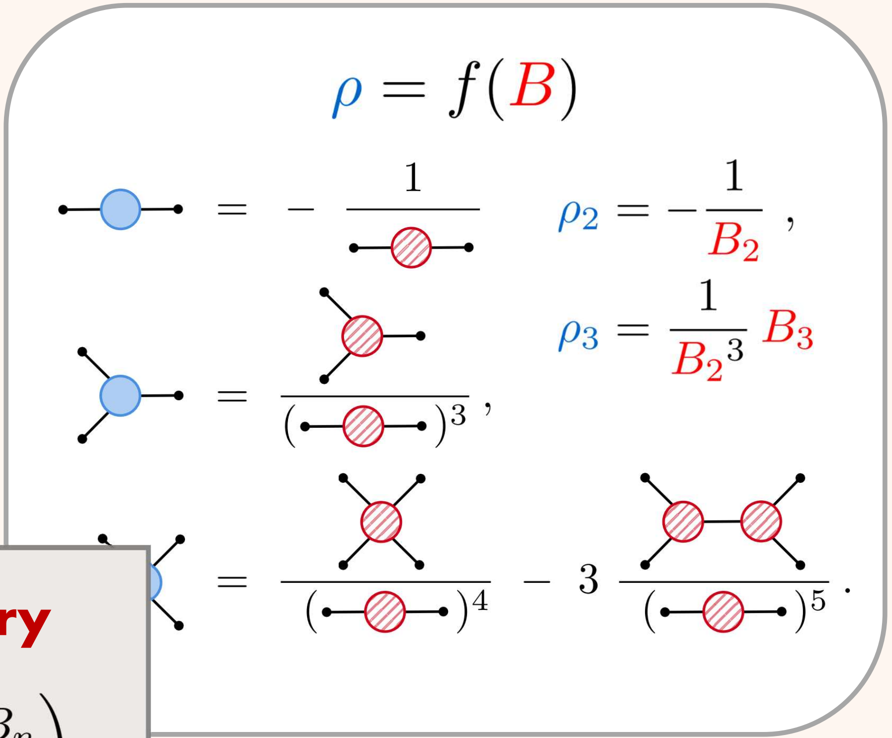


Duality Relation



$$B = f(\rho)$$

Inverse it!



$$\rho = f(B)$$

$$\rho_2 = -\frac{1}{B_2}\ ,$$

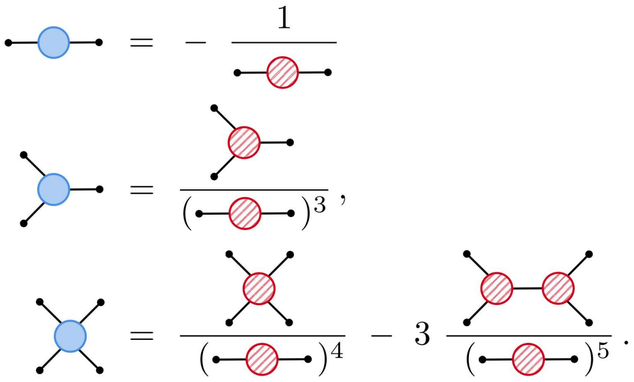
$$\rho_3 = \frac{1}{{B_2}^3} B_3$$

Hidden symmetry

$$g : \begin{pmatrix} \rho_n \\ B_n \end{pmatrix} \mapsto (-i)^n \begin{pmatrix} B_n \\ \rho_n \end{pmatrix}$$

$$g^4 = 1\ ,\ \ g \in \mathbb{Z}_4$$

$$\rho = f(B)$$



Parity

Two facts about parity-odd (scalar) correlators

- It is purely imaginary
- At least 4-pts correlators

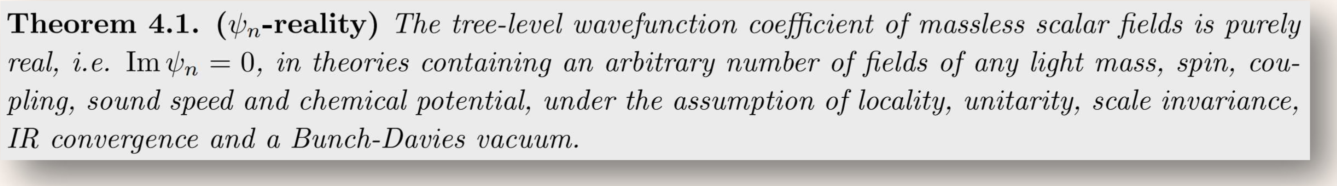
$$\rho_n = \frac{1}{{B_2}^n} \sum_{k=0}^{n-3} (-1)^k \Big(k\text{-cuts} \Big)_B$$

Symmetry

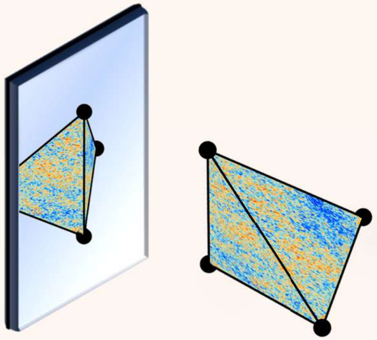
Fundamental Laws

$$\rho_n^{\mathrm{PO}}(\{\mathbf{k}\}) = 0$$

Reality Thereom



Theorem 4.1. **(ψ_n-reality)** *The tree-level wavefunction coefficient of massless scalar fields is purely real, i.e.* $\operatorname{Im}\psi_n = 0$*, in theories containing an arbitrary number of fields of any light mass, spin, coupling, sound speed and chemical potential, under the assumption of locality, unitarity, scale invariance, IR convergence and a Bunch-Davies vacuum.*



Stefanyszyn ,Tong, Y.Z., [2309.07769]

- Unitarity & locality
- BD vacuum
- Tree level
- Scale invariance
- IR convergence

$$\operatorname{Im}\psi \ = \ 0$$

$$\rho_n(\{\mathbf{k}\}) = \psi_n(\{\mathbf{k}\}) + \psi_n^*(\{-\mathbf{k}\})$$

For the parity-odd case:

$$\rho_n^{\mathrm{PO}}(\{\mathbf{k}\}) = \psi_n(\{\mathbf{k}\}) - \psi_n^*(\{\mathbf{k}\})$$

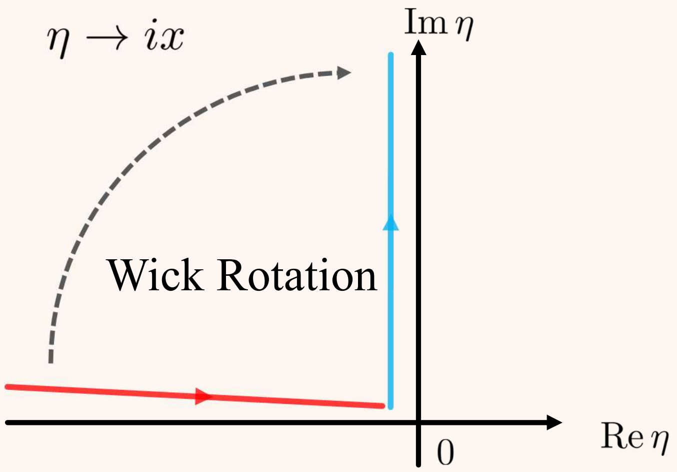
$$\rho_n^{\mathrm{PO}}(\{\mathbf{k}\}) \ = \ 0$$

Reality Thereom

$$\psi_n = \int_{-\infty(1-i\epsilon)}^{0} \left[\prod_{v=1}^{V} d\eta_v \, i\lambda_v \, D_v\right] \left[\prod_{e=1}^{n} K_e\right] \left[\prod_{e'=1}^{I} G_{e'}\right]$$

$$\mathcal{L}_N = \lambda \, a^{1-2m-n} \epsilon^{(3)} \partial_i^{3+2m} \partial_\eta^n \zeta^M \sigma^{N-M}$$

$$\eta \to ix$$



Bulk to boundary propagators: BD vacuum

(massless fields)

$$K \sim (1 - ik\eta)e^{ik\eta} \qquad (1 + kx)e^{-kx} \in \mathbb{R}$$

Vertex: Scale invariant + Unitarity

$$D \sim \epsilon^{(3)} \eta^{n+2m-1} (ik)^{3+2m} \partial_\eta^n \longrightarrow i^{n+2m-1} i^{3+2m} i^n$$

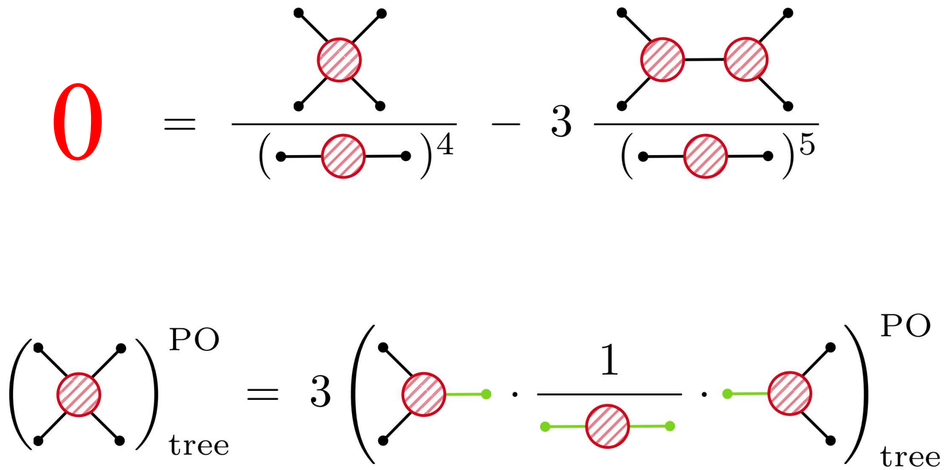
$$\in \mathbb{R}$$

Bulk to bulk propagators:

$$G(k, ix_1, ix_2) \qquad \in \mathbb{R}$$

CCF

$$\rho = f(B)$$



Correlator-to-correlator factorisation (CCF)

The First Example

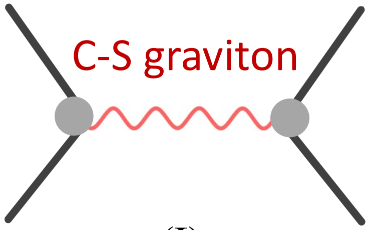
Chern-Simons gravity:

Parity violation

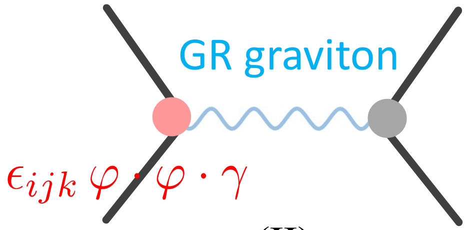
$\phi = \phi_0 + \varphi$

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) + \frac{M_{\rm pl}^2}{2}R - \frac{\phi}{4f}R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\right]$$

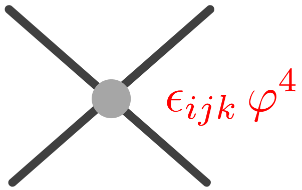
Three possible contributions of PO trispectrum:



(I)

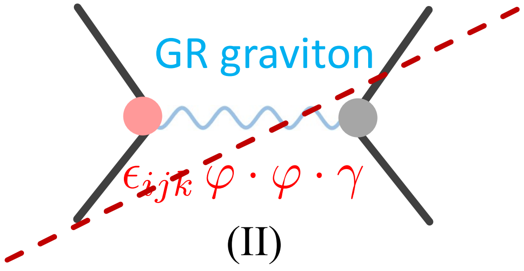


(II)

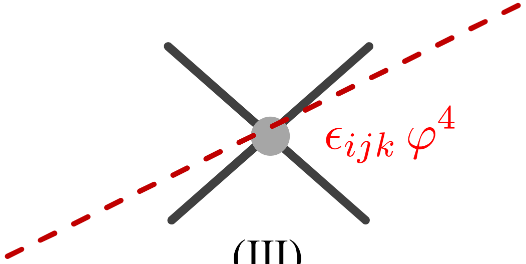


(III)

The First Example



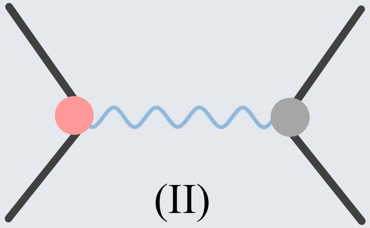
(II)



(III)

No-go theorem:

Parity odd is absent at *tree level*, under the assumption:
(1) All massless dS mode functions ✓
(2) BD vacuum ✓
(3) IR convergence ?



(II)

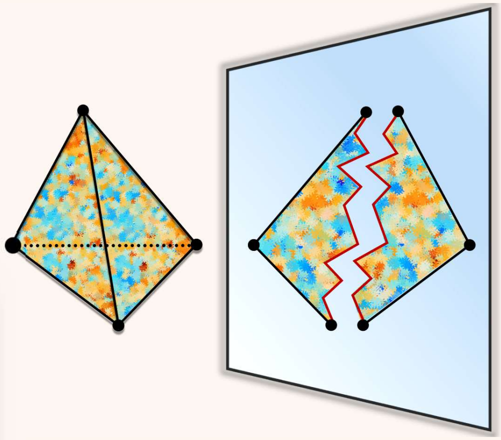
One possible IR divergent coupling

$$a(\eta)\epsilon_{ijk}\,\partial_i\varphi\partial_m\partial_j\varphi\,\gamma_{km}$$

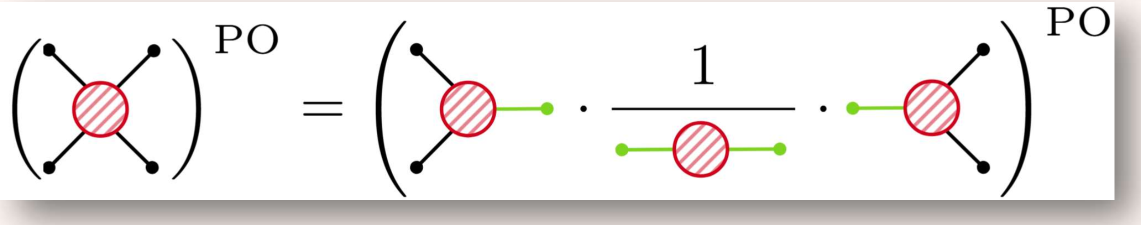
$$\int_\infty^{\eta_0} d\eta\,\frac{1}{\eta}\times\mathcal{O}(1)$$

NO such term in CS Gravity

Summary



CCF Factorisation



CS-gravity example

$$B^{\rm PO}_{\varphi\varphi\varphi\varphi} = -\, i\,\frac{\pi\kappa H^7}{16 M_{\rm pl}^2}\,\frac{[\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_3 - (\hat{\mathbf{k}}_1\cdot\hat{\mathbf{s}})(\hat{\mathbf{k}}_3\cdot\hat{\mathbf{s}})]}{(k_1k_2k_3k_4)^2}\,\frac{\hat{\mathbf{s}}\cdot(\hat{\mathbf{k}}_2\cdot\hat{\mathbf{k}}_4)}{s^3E_L^2E_R^2}$$

$$\times\left(E_L^3 - E_L\, e_2 - e_3\right)\left(E_R^3 - E_R\,\tilde{e}_2 - \tilde{e}_3\right)$$