Stealth black holes in AeST

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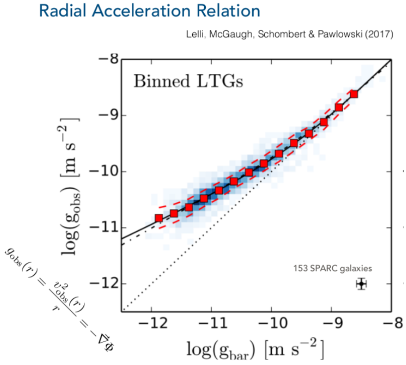
Based on 2412.15395 with Constantinos Skordis

Structure

1. MOND
2. AeST
3. Black holes
4. Conclusion

Hints at MOND

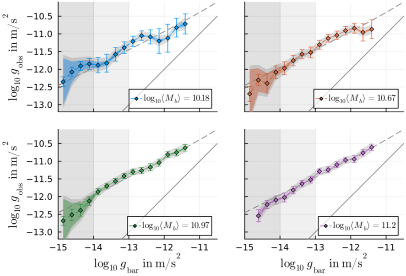


Radial Acceleration Relation

Lelli, McGaugh, Schombert & Pawlowski (2017)

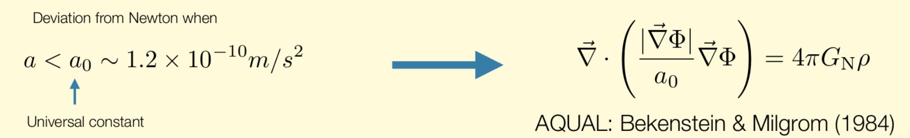
$$g_{\rm obs}(r) = \frac{v_{\rm obs}^2(r)}{r} = -\vec{\nabla}\Phi$$

$$g_{\rm bar}(r) = \frac{v_{\rm bar}^2(r)}{r} = -\vec{\nabla}\Phi_{\rm bar}$$



Mistele et. al. (2024)

AQUAL



Deviation from Newton when

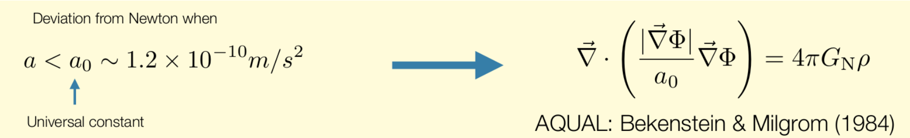
$$a < a_0 \sim 1.2 \times 10^{-10} m/s^2$$

Universal constant

$$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_{\rm N} \rho$$

AQUAL: Bekenstein & Milgrom (1984)

AQUAL



Deviation from Newton when

$$a < a_0 \sim 1.2 \times 10^{-10} m/s^2 \longrightarrow \vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_{\mathrm{N}} \rho$$

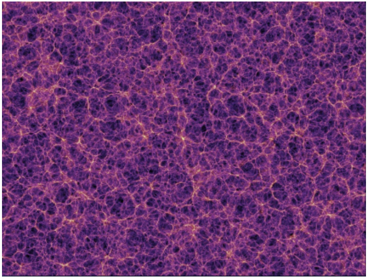
Universal constant

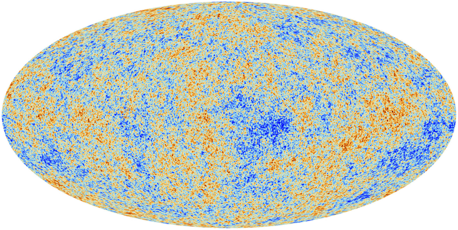
AQUAL: Bekenstein & Milgrom (1984)

- Nonrelativistic - not useful for cosmology
- Cluster conundrum - still need 20% DM content [Aguirre 01]
- Observationally ruled out on smaller scales
    - External field effect on Saturn [Desmond et al 24]
    - Comets and outer solar system constraints [Vokrouhlicky et al 24]

Correct Observables

- Relativistic description
- GW propagate at light speed
- Correct lensing $\rightarrow \Phi = -\Psi$
- CMB and power spectrum: dust $\rightarrow \rho \sim 1/a^3$
- Effective AQUAL at galaxy scales
- Screening of MOND effects at Solar system scales





Aether scalar tensor theory

$$S=\int d^4x\frac{\sqrt{-g}}{16\pi\tilde{G}}\left[R-\frac{K_B}{2}F^{\mu\nu}F_{\mu\nu}+2(2-K_B)J^{\mu}\nabla_{\mu}\phi-(2-K_B)\mathcal{Y}-\mathcal{F}(\mathcal{Y},\mathcal{Q})-\lambda\left(A^{\mu}A_{\mu}+1\right)\right]+S_m[g]$$

where $J_\nu = A^\mu\nabla_\mu A_\nu$, $\quad\mathcal{Y} = q^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, $\quad\mathcal{Q} = A^\mu\nabla_\mu\phi$

Free function $\mathcal{F}$:

- Effective AQUAL with terms as $\mathcal{Y}^{3/2}/a_0$
- Consistent Newtonian limit + correct cosmology

$$\mathcal{F} = (2 - K_{\mathrm{B}})\,\lambda_s\mathcal{Y} - 2\mathcal{K}_2\,(\mathcal{Q}-\mathcal{Q}_0)^2 + \ldots, \tag{1}$$

Spherically symmetric static solutions

Spherically symmetric static ansatz

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Psi}dr^2 + r^2 d\Omega, \tag{2}$$

where $\Phi = \Phi(r)$ and $\Psi = \Psi(r)$.

$$\phi = \mathcal{Q}_0(qt + R(r)) \tag{3}$$
$$A_r = A(r) \tag{4}$$

- $q = 1$ are physical - consistently attachable to FLRW solution $\phi = \mathcal{Q}_0 t$
- No unphysical field divergences (does $J^\mu J_\mu$ divergence matter?)
- Asymptotically flat
- No naked singularities

Shift symmetry generates Noether current and charge

$$\phi \to \phi + c \implies \nabla_\mu J^\mu = 0$$

In $q = 1$ case, $Q_\phi = 0$. In general, there is additional integration constant φ_0.

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Neglecting cosmological terms $\mathcal{K}_2 << 1$, we get from field equations

$$\boxed{\Phi = -\Psi} \tag{5}$$

which simplifies calculations.

$q_s = 1$ solutions

Reissner-Nordstrom metric:

$$\boxed{e^{2\Phi} = 1 - \frac{2G_{\mathrm{N}}M}{r} + \frac{nq_A^2}{r^2}} \tag{6}$$

stealth black hole with secondary hair

$$A = s_A e^{-2\Phi}\sqrt{\left(1 + \frac{|q_A|}{r}\right)^2 - e^{2\Phi}} \tag{7}$$

$$R' = -\frac{1}{1 + \frac{|q_A|}{r}}\left[\frac{1}{(1+\lambda_s)\mathcal{Q}_0}\frac{q_A}{r^2} + \epsilon_A A\right] \tag{8}$$

Divergent at horizon, but this is only a coordinate effect

$q_s = 0$ solutions

Reissner Nordstrom metric again with fields

$$A = s_A e^{-2\Phi}\sqrt{\frac{2(G_{\mathrm{N}}M + q_A)}{r} + \frac{q_{BH}^2(1-\tilde{n})}{\tilde{n}}\frac{1}{r^2}} \tag{9}$$

$$(1+\lambda_s)\phi = \ln\sqrt{1 + \frac{2q_A}{r} + \frac{q_{BH}^2}{\tilde{n}}\frac{1}{r^2}} + \frac{s_{\varphi_0}}{2\tilde{m}}\ln\frac{1 + \frac{q_A + \tilde{m}|\tilde{\varphi}_0|}{r}}{1 + \frac{q_A - \tilde{m}|\tilde{\varphi}_0|}{r}} \tag{10}$$

- Possibly diverging Noether charge with $\varphi_0 \neq 0$
- $\tilde{n} = \frac{2+K_B\lambda_s}{2(1+\lambda_s)}, \tilde{m} = \sqrt{\frac{2-K_B}{2+K_B\lambda_s}},$

Peculiar solutions with $A = 0$

$$\boxed{ds^2 = -\left(\frac{|u-u_1|}{|u-u_2|}\right)^{\frac{\tilde{n}}{n}} dt^2 + \frac{r_0^2}{u^2}\frac{|u-u_1|^{\frac{\tilde{n}(1+n)}{n}}}{|u-u_2|^{\frac{\tilde{n}(1-n)}{n}}}\left[\frac{du^2}{u^2(u-u_1)(u-u_2)} + d\Omega\right]} \tag{11}$$

- Naked singularities
- $(--++)$ signature solutions
- Cosmological solutions with nonzero $\mathcal{K}_2$:
  Reissner-Nordstrom-deSitter metric with hair

$$e^{2\Phi} = 1 - \frac{2G_{\mathrm{N}}M}{r} + \frac{nq_A^2}{r^2} + \Lambda r^2 \tag{12}$$

Not considering mond regime

Elling-Jacobson Wormholes

$$e^{2\Phi} = \left(\frac{1 - Y/Y_-}{1 - Y/Y_+}\right)^{\frac{-Y_+}{2+Y_+}} \tag{13}$$

$$\frac{r_{\min}}{r} = \left(\frac{Y}{Y - Y_-}\right)\left(\frac{Y - Y_-}{Y - Y_+}\right)^{\frac{1}{2+Y_+}} \tag{14}$$

- Known solutions in Einstein-Aether
- One branch of our solutions cover this manifold
- Additional scalar hair

Frames at infinities

- In principle, the field frames do not have to coincide
- EOM force them to coincide at $r \to \infty$.
- Consequence of spherically symmetric ansatz

Conclusions

- AeST admits consistent **stealth black holes** with nontrivial vector and scalar hair
- Other BH solutions with problems of cosmological connectivity
- Peculiar wormhole solutions
- Since these are the most general SSS solutions of the theory - basis for stellar solutions

Further on

- Quasinormal modes and stability
- Thermodynamics
- Weak field regimes of khronometric theories

Thank you!