SEMICLASSICAL GRAVITY AND BEYOND

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- L. Liu and T. Prokopec,``Appearances are deceptive: Can graviton have a mass?,'' [arXiv:2407.12657 [hep-th]] + in prep.
- Duojie Jimu, T. Prokopec 2410.01449 [gr-qc], JHEP (2025)
- R. Kavanagh, S. Leisink, T. Prokopec, in preparation (2025)
- A. Fennema and T. Prokopec, in preparation (2025)
- S. Park, T. Prokopec, R.P. Woodard, ``Quantum Scalar Corrections to the Gravitational Potentials on de Sitter Background,'' JHEP **01** (2016), 074 doi:10.1007/JHEP01(2016)074 [arXiv:1510.03352 [gr-qc]].

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SEMICLASSICAL GRAVITY

CLASSICAL GRAVITY

- EINSTEIN'S EQUATION: $G_{\mu\nu}(g) = 8\pi G T_{\mu\nu}(g, \psi_i)$

- $\kappa^2 \equiv 16\pi G$ = gravitational coupling constant (G=Newton constant)
- $G_{\mu\nu}(g)$ = Einstein curvature tensor, $g_{\mu\nu}$ metric tensor
- $T_{\mu\nu}(g, \psi_i)$ = Energy-momentum tensor, ψ_i = matter fields

EXAMPLE COSMOLOGY:

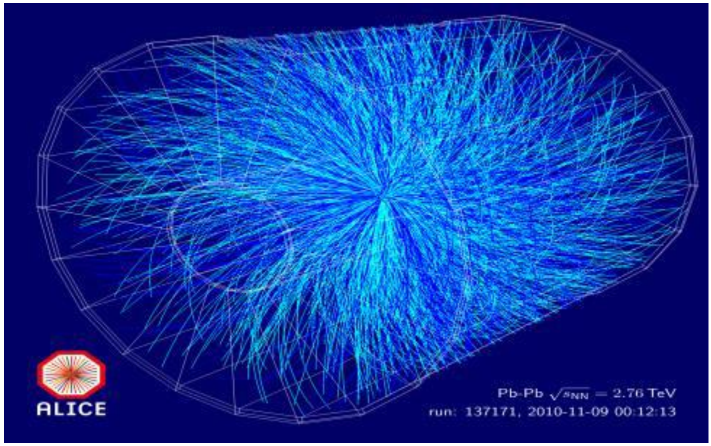
- PERFECT FLUID FORM: $T_{\mu\nu} = (P + \rho) u_\mu u_\nu + P g_{\mu\nu}$,

- P, ρ, u^μ = fluid pressure, energy density and 4-velocity ($g_{\mu\nu} u^\mu u^\nu$ = -1).
- $P = P(\rho) = w\rho$ = equation of state.

In fluid rest frame: FRIEDMANN EQUATIONS: $H^2 = \frac{8\pi G}{3}\rho,\ \dot{H} = -\frac{4\pi G}{3}(P + \rho)$.

Plus the conservation law: $\nabla^\mu T_{\mu\nu} = 0 \Rightarrow \dot{\rho} + 3H(P + \rho) = 0$.

- POTENTIAL PROBLEM: EVEN THOUGH GRAVITY MAY BE TREATED IN MOST CASES AS CLASSICAL, MATTER IS KNOWN TO BE QUANTUM, AND QUANTUM EFFECTS OF MATTER BECOME STRONGER AT HIGHER ENERGIES, WHICH ARE PROBED IN ACCELERATORS AND IN THE PRIMORDIAL UNIVERSE.



Pb-Pb COLLISION AT ALICE (~TeV) [ALICE/CERN]

SEMICLASSICAL GRAVITY

SEMICLASSICAL GRAVITY (SCG), BASIC IDEA:

- SINCE EFFECTS OF QUANTUM GRAVITY BECOME IMPORTANT ONLY AT THE PLANCK SCALE, IN ACCORDANCE WITH THE FUNDAMENTAL ASSUMPTION OF EFTs (REGARDING SEPARATION OF SCALES), WE CAN TREAT MATTER AS QUANTUM AND GRAVITY AS CLASSICAL, THUS OVERCOMING THE SHORTCOMINGS OF THE CLASSICAL DESCRIPTION.

- THE FUNDAMENTAL EQUATION OF SCG:

$$G_{\mu\nu}(g) = 8\pi G\left(T_{\mu\nu}(g,\psi_i) + \left\langle \delta\hat{T}_{\mu\nu}(g,\psi_i)\right\rangle\right)$$

- NB: the classical $T_{\mu\nu}(g,\psi_i)$ and quantum $\left\langle \delta\hat{T}_{\mu\nu}(g,\psi_i)\right\rangle$ energy-momentum tensors are separately conserved.

- NB2: $g_{\mu\nu}$ IS THE METRIC OF SCG, i.e. IT CONTAINS BOTH CLASSICAL AND QUANTUM RESPONSE:

$$g_{\mu\nu} \equiv \left\langle \hat{g}_{\mu\nu}(g,\psi_i)\right\rangle = g^{cl}_{\mu\nu} + \delta g^{q}_{\mu\nu},$$

SUCH THAT ONE CAN APPLY PERTURBATIVE TECHNIQUES OF QFT TO $\delta g^{q}_{\mu\nu}$, i.e. ONE CAN DISTINQUISH DIFFERENT LOOP CONTRIBUTIONS: $\delta g^{q}_{\mu\nu} = \delta g^{(1)}_{\mu\nu} + \delta g^{(2)}_{\mu\nu} + ..$

SEMICLASSICAL GRAVITY 2

- IN 1970s SEMICLASSICAL GRAVITY (SCG, ALSO KNOWN AS QUANTUM BACKREACTION) WAS EXTENSIVELY STUDIED.

- ‘t HOOFT AND VELTMAN (1974) USED SEMICLASSICAL GRAVITY TO PROVE THAT GRAVITY (COUPLED TO MATTER) WAS NON-RENORMALIZABLE AT 1-LOOP; PURE GRAVITY IS NON-RENORMALIZABLE AT 2-LOOPS (Goroff, Sagnotti, 1981).

- BIRRELL AND DAVIES’ TEXTBOOK (Quantum Fields in Curved Space, 1982) STUDY EXTENSIVELY RENORMALIZATION OF THE ENERGY-MOMENTUM TENSOR, BOTH ON FLAT AND CURVED BACKGROUNDS. RESULT: ONE CAN USE e.g. SCHWINGER-DEWITT PROPER-TIME REPRESENTATION OF THE PROPAGATORS, TO ISOLATE UV DIVERGENCES GENERATED BY THE QUANTUM MATTER FLUCTUATIONS ON GENERAL CURVED BACKGROUNDS. THE IR CONTRIBUTION REMAINS UNKNOWN HOWEVER, AND IT IS KNOWN TODAY ONLY IN A HANDFUL OF SIMPLE EXAMPLES, SUCH AS DE SITTER AND ANTI-DE SITTER SPACES.

- THE PRINCIPAL RESULT: THE STRUCTURE OF UV DIVERGENCES AT 1-LOOP IS LOCAL, AND CAN BE EXPRESSED IN TERMS OF THE FOLLOWING 5 CURVATURE INVARIANTS:

$$H_{\mu\nu}(x) \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)}\int \sqrt{-g}\, R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \qquad G_{\mu\nu}(x) \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)}\int \sqrt{-g}\, R, \qquad g_{\mu\nu}(x) \equiv -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)}\int \sqrt{-g},$$

$$H^{(1)}_{\mu\nu}(x) \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)}\int \sqrt{-g}\, R^2,$$

$$H^{(2)}_{\mu\nu}(x) \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}(x)}\int \sqrt{-g}\, R_{\alpha\beta}R^{\alpha\beta},$$

- IN D = 4 THE 3 ARE LINEARLY DEPENDENT [GAUSS-BONNET TOPOLOGICAL INVARIANT]:

$$H_{\mu\nu} + H^{(1)}_{\mu\nu} - 4\, H^{(2)}_{\mu\nu} = O(D-4)$$

WEYL ANOMALY

- WEYL (OR CONFORMAL) ANOMALY [Capper & Duff, 1974-75; Christensen & Duff, 1978-80]

- THEORIES THAT ARE CLASSICALLY WEYL (CONFORMALLY) INVARIANT,

$$g^{\mu\nu}T_{\mu\nu} = 0,$$

BECOME ANOMALOUS AT THE QUANTUM (LOOP) LEVEL,

$$g^{\mu\nu}\langle\delta\hat{T}_{\mu\nu}(g,\psi_i)\rangle = \frac{1}{2880\pi^2}\left(\alpha R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} + \beta\left(R_{\alpha\beta}R^{\alpha\beta} - \frac{1}{3}R^2\right) + \gamma\,\Box R + \delta R^2\right)$$

SPIN	α	β	δ
SCALAR:	-1	-1	$-90\left(\xi - \frac{1}{6}\right)^2$
DIRAC:	$-\frac{7}{2}$	-11	0
VECTOR:	-33	27	$-\frac{5}{2}$
GRAVITON:	-189	x	$-\frac{747}{4}$

- γ IS NON-UNIVERSAL (DEPENDS ON THE RENORMALIZATION SCHEME USED).

- THE ORIGIN OF WEYL ANOMALY: IN DIM REG, THE COUNTERTERM COEFFICIENTS $\sim \frac{\mu^{D-4}}{D-4}$ INTRODUCE A DEPENDENCE ON ARBITRARY SCALE μ TO THE EFFECTIVE ACTION AND ENERGY-MOMENTUM TENSOR, THUS BREAKING SCALE INVARIANCE OF THE THEORY. THE PROBLEM CAN BE FIXED BY GAUGING THE GLOBAL SCALING SYMMETRY, i.e. BY INTRODUCING A WEYL VECTOR FIELD W_μ, SUCH AS A TORSION TRACE: $W_\mu \sim T^{\nu}_{\mu\nu}$. THE ANOMALY BECOMES A WARD IDENTITY FOR THE DILATION CURRENT Π^μ:

$$\nabla_\mu\langle\hat{\Pi}^\mu(x)\rangle = -\,g^{\mu\nu}\,\langle\hat{T}_{\mu\nu}(x)\rangle$$

[Lucat, Prokopec, 2017]

SEMICLASSICAL GRAVITY: A SUMMARY

- AFTER RENORMALIZATION, THE EOM OF SCG BECOMES:

$$G_{\mu\nu}(g) = 8\pi G^r \left(T_{\mu\nu}(g,\psi_i) + \langle \delta\hat{T}_{\mu\nu}(g,\psi_i)\rangle_{\text{non-local}} + \alpha H_{\mu\nu}(x) + \beta H^{(1)}_{\mu\nu}(x) + \gamma H^{(2)}_{\mu\nu}(x)\right) + \Lambda^r g_{\mu\nu}$$

- COEFFICIENTS $G^r, \Lambda^r, \alpha, \beta, \gamma$ ARE THE 5 RENORMALIZED GRAVITATIONAL COUPLINGS WHICH OUGHT TO BE FIXED BY EXPERIMENTS. OF COURSE, G^r, Λ^r HAVE BEEN FIXED!
- α, β, γ RENDER THE THEORY NONRENORMALIZABLE, AND WORSE NON-UNITARY (AS AN EFFECTIVE THEORY, $\propto(\alpha + \frac{\gamma}{4})$ IN THE THEORY EXHIBITS AN OSTROGRADSKY INSTABILITY)
- ANY CLASSICAL DESCRIPTION WOULD BE CUTOFF DEPENDENT. HERE DEP. ON: $\alpha, \beta, (\gamma)$.

- UNLESS α, β, γ ARE HUGE, CONTRIBUTIONS OF THE LATTER TERMS ARE SMALL, AS THEY ARE SUPPRESSED AS $(\alpha, \beta, \gamma)\times \kappa^2 E^2 \sim (\alpha, \beta, \gamma) \times \frac{E^2}{M_{\text{Pl}}^2}$, WHERE $M_{\text{Pl}} = \frac{1}{\sqrt{8\pi G}} \sim 2.4 \times 10^{18}\,\text{GeV}$ IS THE REDUCED PLANCK MASS (ENERGY), AND THEREFORE THEY CAN BE TREATED PERTURBATIVELY, AVOIDING THUS OSTROGRADSKY INSTABILITIES.

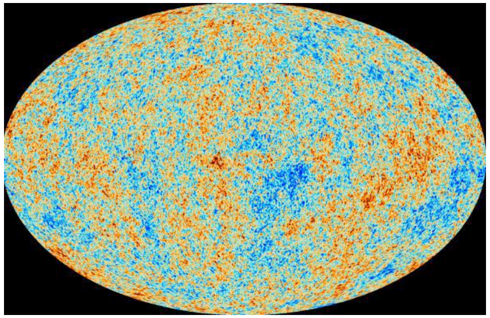
- NB: β CAN BE LARGE, AS IN STAROBINSKY INFLATION, AS IT DOES NOT GENERATE OSTROGRADSKY INSTABILITIES. THE REASON: $R + R^2$ THEORY OF GRAVITY IS ACTUALLY A SCALAR-TENSOR THEORY, WHICH IS STABLE.

- NB2: APPLICATIONS OF SCG HAVE BEEN VERY SUCCESSFUL IN COSMOLOGY, AND APPROXIMATING $\langle \delta\hat{T}_{\mu\nu}(g,\psi_i)\rangle_{\text{non-local}}$ BY 1 OR 2 LOOP THERMAL MATTER CONTRIBUTIONS FROM STANDARD MODEL MATTER HAS BEEN A STANDARD WAY OF DESCRIBING THE BIG BANG MODEL. HOWEVER, THIS CONTAINS A VERY LIMITED INFORMATION ABOUT THE EARLY UNIVERSE, AS IT DESCRIBES THE DYNAMICS OF ONE DEGREE OF FREEDOM.

GRAVITATIONAL PERTURBATIONS

NEED TO GO BEYOND SEMICLASSICAL GRAVITY

- IN THE 20TH CENTURY WE HAD VERY LITTLE OBSERVATIONAL INFORMATION THAT GOES BEYOND THAT OF THE BACKGROUND EVOLUTION, DESCRIBED BY THE HUBBLE PARAMETER $H = H(t)$.
- THAT HAS DRAMATICALLY CHANGED IN THE 21ST CENTURY: BY MEASUREMENTS OF CMB FLUCTUATIONS (WMAP, Planck, Simons Observatory, EUCLID) AND GRAVITATIONAL WAVES (GW); WE STILL DO NOT HAVE A CONVINCING DETECTION OF PRIMORDIAL GWs.
- THE AMOUNT OF DATA ABOUT THE PRIMORDIAL UNIVERSE STORED IN <u>GRAVITATIONAL</u> <u>PERTURBATIONS</u> IS ABOUT 10^7(Planck) AND BY THE END OF THE DECADE WE MAY REACH $\sim 10^{10}$ DATA POINTS, WHICH IS TO BE CONTRASTED WITH 1 DATA POINT IN H(t)!



Planck image of the CMB temperature fluctuations [Image credit: ESA/Planck]

- PRESENTLY, THE EVOLUTION OF GRAVITATIONAL PERTURBATIONS (known as <u>COSMOLOGICAL PERTURBATIONS</u>) IS MODELED CLASSICALLY, i.e. THEY ARE ASSUMED TO EVOLVE IN A CLASSICAL BACKGROUND OF MATTER FIELDS. THIS MEANS THAT – AS REGARDING COSMOLOGICAL PERTURBATIONS – WE ARE TREATING THE MATTER BACKGROUND AS AN IDEAL FLUID, THEREBY COMPLETELY <u>IGNORING</u> THE QUANTUM NATURE OF MATTER! AS ARGUED ABOVE, MATTER BECOMES MORE AND MORE QUANTUM AS ONE INCREASES THE ENERGY, AND AT VERY HIGH ENERGIES (VERY EARLY UNIVERSE) THIS APPROXIMATION BECOMES WORSE, AND EVENTUALLY BREAKS DOWN.

- <u>Q</u>: WHAT ARE THE REASONS TO GO BEYOND THE CLASSICAL BACKGROUND?

CLASSICAL EVOLUTION OF GRAVITATIONAL PERTURBATIONS

CLASSICAL EVOLUTION OF COSMOLOGICAL PERTURBATIONS

[Liu, Prokopec https://arxiv.org/pdf/2407.12657]

- CONSIDER MASSIVE DIRAC FERMIONS IN A THERMAL STATE AS THE MATTER THAT DRIVES THE EXPANSION OF THE UNIVERSE:

$$S_{\rm total}[\psi,\bar\psi,g_{\mu\nu}] = S_{\rm Dirac}[\psi,\bar\psi,g_{\mu\nu}] + S_{\rm HE}[g_{\mu\nu}], \qquad S_{\rm HE}[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^Dx\sqrt{-g}\,[R-(D-2)\Lambda]$$

$$S_{\rm Dirac}[\psi,\bar\psi,g_{\mu\nu}] = \int d^Dx\sqrt{-g}\left[\frac{i}{2}\left(\bar\psi e^\mu_b\gamma^b\nabla_\mu\psi - \nabla_\mu\bar\psi e^\mu_b\gamma^b\psi\right) - \bar\psi M\psi\right]$$

- e^μ_b= TETRAD FIELD, $\quad g_{\mu\nu} = e^a_\mu\eta_{ab}e^b_\nu,$

$$\nabla_\mu\psi = \partial_\mu\psi - \Gamma_\mu\psi, \qquad \nabla_\mu\bar\psi = \partial_\mu\bar\psi + \Gamma_\mu\bar\psi, \qquad \Gamma_\mu = \frac{1}{8}e^\nu_c\left(\partial_\mu e_{\nu d} - \Gamma^\varrho_{\mu\nu}e_{\rho d}\right)[\gamma^c,\gamma^d].$$

- EXPAND THE ACTION AROUND A (FLAT) COSMOLOGICAL BACKGROUND TO SECOND ORDER IN GRAVITATIONAL PERTURBATIONS $h_{\mu\nu}(x)$:
  $g_{\mu\nu} = a^2\left(\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\right)$, $\chi = a^{(D-1)/2}\psi$, $\bar\chi = a^{(D-1)/2}\bar\psi$:

$$S^{(2)}_{\rm HE}[g_{\mu\nu}] \supset \int d^Dx\, a^{D-2}\left\{\left(h^2 - 2h^{\mu\nu}h_{\mu\nu}\right)\frac{D-2}{8}\left((D-3)H^2 + 2H' - a^2\Lambda\right) + \left(hh^{\mu\nu} - 2h^{\mu\varrho}h^\nu_\varrho\right)\delta^0_\mu\delta^0_\nu\frac{D-2}{2}(H'-H^2)\right\}$$

$$S^{(2)}_{\rm Dirac}[g_{\mu\nu}] \supset \int d^Dx\left\{\left(h^2 - 2h^{\mu\nu}h_{\mu\nu}\right)L - \left(2hh^{\mu\nu} - 3h^{\mu\varrho}h^\nu_\varrho\right)K_{\mu\nu}\right\},\ K_{\mu\nu} \equiv \frac{i}{2}\left(\bar\chi\gamma_\nu\partial_\mu\chi - (\partial_\mu\bar\chi)\gamma_\nu\chi\right),\ L = \eta^{\mu\nu}K_{\mu\nu} - a\bar\chi M\chi$$

- ONE CAN SHOW THAT THESE DO NOT VANISH UPON (SEMI-)CLASSICAL EQUATIONS OF MOTION (SCG) ARE USED, SUGGESTING THAT THE GRAVITON $h_{\mu\nu}(x)$ HAS A MASS!

[Rick S. Vinke, UU Theses repository (2020), https://studenttheses.uu.nl/handle/20.500.12932/36948]

DYNAMICAL GRAVITON MASS?

- ANALYSING THE RESULTING EQUATIONS OF MOTION FOR $h_{\mu\nu}$ REVEALS:

$$\mathrm{LHS} = L_{\mu\nu}{}^{\varrho\sigma} h_{\varrho\sigma} - \frac{D-2}{4} a^2 \Lambda (h\eta_{\mu\nu} - 2h_{\mu\nu}) \supset -\frac{D-2}{2}\left((D-3)H^2 + 2H'\right)(h_{\mu\nu} + h_{00}\eta_{\mu\nu}) + \frac{D-2}{2} a^2 \Lambda (h_{\mu\nu} - \tfrac{1}{2} h\eta_{\mu\nu})$$

$$\mathrm{RHS} \supset 8\pi G a^{2-D} h_{\mu\nu} \left(L^{(0)} + \tfrac{1}{2} Q^{(0)}\right) \xrightarrow{L^{(0)}=0} 8\pi G a^2 h_{\mu\nu} \tfrac{1}{2} P^{(0)}$$

WHERE THE BACKGROUND EQUATIONS OF MOTION ARE:

$$G^{(0)}_{\mu\nu} + \frac{D-2}{2} a^2 \eta_{\mu\nu}\Lambda = 8\pi G a^{2-D} \left(L^{(0)} \eta_{\mu\nu} - K^{(0)}_{\mu\nu}\right) = 8\pi G\left[\left(P^{(0)} + \rho^{(0)}\right) a^2 \delta^0_\mu \delta^0_\nu + a^2 \eta_{\mu\nu} P^{(0)}\right]$$

$$K^{(0)}_{\mu\nu} = -Q^{(0)} \eta_{\mu\nu} - \delta^0_\mu \delta^0_\nu S^{(0)} \Rightarrow P^{(0)} = \tfrac{1}{a^D}\left(L^{(0)} + Q^{(0)}\right), \quad P^{(0)} + \rho^{(0)} = \tfrac{1}{a^D} S^{(0)}$$

$$G^{(0)}_{\mu\nu} + \tfrac{D-2}{2} a^2 \eta_{\mu\nu} \Lambda = -\tfrac{D-2}{2} \eta_{\mu\nu}\left[(D-3)H^2 + 2H' - a^2\Lambda\right] - (D-2)\delta^0_\mu \delta^0_\nu (H' - H^2)$$

- COMPARING TERMS $\propto \eta_{\mu\nu}$ REVEALS THAT ZERO-DERIVATIVE TERMS $\propto h_{\mu\nu}$ DO NOT CANCEL: $\mathrm{LHS} = 8\pi G a^2 h_{\mu\nu} P^{(0)}$, $\mathrm{RHS} = 8\pi G a^2 h_{\mu\nu} \frac{1}{2} P^{(0)}$, i.e. A FACTOR $\frac{1}{2}$ SURVIVES, SUGGESTING $8\pi G \frac{1}{2} P^{(0)}$ $(a^2 h_{\mu\nu})$ AS THE DYNAMICAL GRAVITON MASS TERM, i.e. $m_g^2 = 8\pi G \frac{1}{2} P^{(0)}$.

[Rick S. Vinke, UU Theses repository (2020), https://studenttheses.uu.nl/handle/20.500.12932/36948]

INTERMEDIATE SUMMARY

- DYNAMICAL GRAVITONS (in harmonic gauge $h_{00} = 0 = h$) OBEY THE EQUATIONS OF MOTION:

$$\left(\Box_S + m_g^2\right)h_{\mu\nu} = 0,$$

WHERE THE MASS $m_g^2 = 8\pi G a^2 \frac{1}{2} P^{(0)} \xrightarrow{D\to 4,\, \Lambda\to 0} -\left(\dot{H} + \frac{3}{2}H^2\right)$ IS NOT SMALL.
THE WOULD-BE GRAVITON MASS PARAMETER IS POSITIVE (NEGATIVE) IN DECELERATING (ACCELERATING) SPACETIMES.

- FERMIONS DO NOT COUPLE TO VECTOR GRAVITATIONAL PERTURBATIONS, THE NONDERIVATIVE CONTRIBUTIONS TO THE TWO SCALARS h_{00}, h DO NOT VANISH.
- IF TRUE, THAT WOULD POSE A PROBLEM FOR SCALAR COSMOLOGICAL PERTURBATIONS CREATED DURING INFLATION, AS THE BIG BANG PLASMA WOULD DAMP THEM.

- WHAT HAVE WE FORGOTTEN? IS THERE A SYMMETRY?

- WE KNOW THAT (PERTURBATIVE) MASS FOR GAUGE BOSONS IS FORBIDDEN BY GAUGE SYMMETRY. GRAVITONS ALSO OBEY A GAUGE SYMMETRY!

NOETHER THEOREM

- THE GAUGE SYMMETRY UNDER LINEAR COORDINATE TRANSFORMATIONS, ($x^\mu \to x^\mu + \xi^\mu(x)$ THE METRIC TRANSFORMS AS, $g_{\mu\nu} \to g_{\mu\nu} - L_\xi g_{\mu\nu} = g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$.

- VARYING THE GRAVITATIONAL ACTION PRODUCES:

$$\delta S_{\rm HE}[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^Dx\sqrt{-g}\left[G_{\mu\nu} + \frac{D-2}{2}\Lambda g_{\mu\nu}\right]\delta g^{\mu\nu} \to \frac{2}{16\pi G}\int d^Dx\sqrt{-g}\left[G_{\mu\nu} + \frac{D-2}{2}\Lambda g_{\mu\nu}\right]\nabla^\mu \xi^\nu$$

$$\to -\frac{2}{16\pi G}\int d^Dx\sqrt{-g}\left[\nabla^\mu G_{\mu\nu} + \frac{D-2}{2}\Lambda \nabla^\mu g_{\mu\nu}\right]\xi^\nu(x)$$

- SINCE $\xi^\nu(x)$, IS ARBITRARY, THIS GIVES: $\nabla^\mu G_{\mu\nu} = 0$, $\nabla^\mu g_{\mu\nu} = 0$, WHICH ARE AUTOMATICALLY SATISFIED BY BIANCHI IDENTITY AND METRIC COMPATIBILITY.

- VARYING THE MATTER ACTION GIVES:

$$\delta S_{\rm Dirac}[\psi, \bar{\psi}, g_{\mu\nu}] = -\int d^Dx\sqrt{-g}\,[T_{\mu\nu}(x)]\,\nabla^\mu \xi^\nu \to \int d^Dx\sqrt{-g}\,[\nabla^\mu T_{\mu\nu}(x)]\,\xi^\nu(x) = 0,$$

IMPLYING THE ENERGY-MOMENTUM CONSERVATION, $\nabla^\mu T_{\mu\nu}(x)$=0. IT IS WORTH CHECKING IT. NOTE THAT $T_{\mu\nu}(x)$ CONTAINS A PART THAT IS 0th ORDER AND A PART LINEAR IN $h_{\mu\nu}(x)$:

$$T_{\mu\nu}(x) = T^{(0)}_{\mu\nu}(x) + T^{(1)}_{\mu\nu}(x) + \left(O(h_{\mu\nu})^2\right)$$

- SIMILARLY, THE DIRAC EQUATION CONTAINS A PART THAT IS 0th ORDER AND A PART LINEAR IN $h_{\mu\nu}(x)$. TAKING ACCOUNT OF THESE, ONE OBTAINS [arxiv:2407.12657]

$$T^{(1)}_{\mu\nu} = \frac{\kappa}{a^{D-2}}\left[h_{\mu\nu}\left(L^{(0)} + Q^{(0)}\right) - \delta^0_{(\mu} h_{\nu)0} S^{(0)} + \frac{1}{2}\eta_{\mu\nu} h_{00} S^{(0)} + \delta^0_\mu \delta^0_\nu S^{(1)}\right],\quad S^{(1)} = \kappa a\int_{\eta_0}^{\eta}\frac{d\eta'}{2a(\eta')}\left[h_{00}\left(\partial_0 S^{(0)}\right) - (\partial_0 h) S^{(0)} - DHh_{00}S^{(0)}\right]$$

- INCLUDING THESE CONTRIBUTIONS, THE DYNAMICAL GRAVITON MASS DISAPPEARS, HOWEVER THE COUPLING TO GRAVITATIONAL SCALARS h_{00}, h & VECTORS h_{0i} PERSISTS.

WHAT HAVE WE LEARNED?

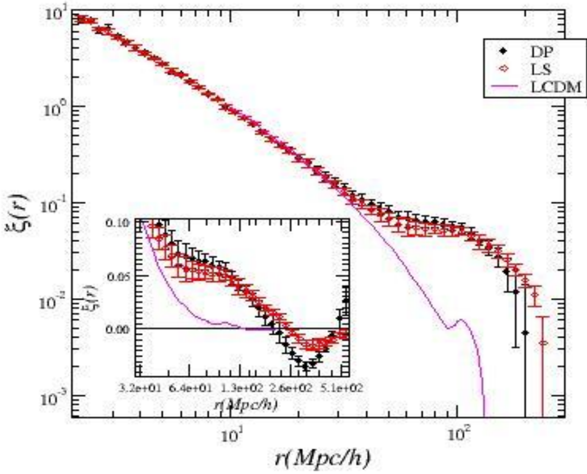
- THE LESSON TO TAKE FROM THIS SIMPLE CONSIDERATION, IS THAT UNLESS WE ARE CAREFUL WITH THE EVOLUTION EQUATIONS FOR GRAVITATIONAL PERTURBATIONS, WE WILL LIKELY REACH WRONG CONCLUSIONS.

- THE QUESTION IS THEN: IS THERE A SYSTEMATIC WAY OF TREATING THE EVOLUTION (AND CREATION) OF GRAVITATIONAL PERTURBATIONS WITHOUT FALLING INTO A PITFALL?

- ANSWER: QFT COMES TO THE RESCUE!

A SYSTEMATIC ORGANISING PRINCIPLE OF HOW TO EVOLVE PERTURBATIONS IN QFT IS THE LOOP EXPANSION. THIS PRINCIPLE HAS ALREADY SUCCESSFULLY BEEN APPLIED TO LSS [Scoccimarro and collaborators, e-Print: astro-ph/0112551 [astro-ph]], WHERE STANDARD (CLASSICAL) PERTURBATION THEORY HAS BEEN USED TO STUDY THE EVOLUTION OF COSMOLOGICAL CORRELATORS. THIS THEORY HAS BEEN REFINED BY AN EFTofLSS [Baumann, Nicolis, Senatore, Zaldarriaga e-Print: 1004.2488 [astro-ph.CO]].

- THE DIFFERENCE BETWEEN THIS AND THE QFT APPROACH IS IN THE FACT THAT CORRELATORS ARE CLASSICAL STOCHASTIC (DENSITY) FIELDS, FOR WHICH OPERATOR ORDERING IS IRRELEVANT. THE MALADY OF CLASICAL APPROACH IS A CUTOFF DEPENDENCE (EFTofLSS EXHIBITS BOTH IR AND UV CUTOFF DEPENDENCE).
- THE ADVANTAGE OF USING QFT METHODS IS THAT ONE CAN GET RID OFF ANY DEPENDENCE ON (IR OR UV) CUTOFFS.



GALAXY-GALAXY 2-POINT FUNCTION [SDSS]

QUANTUM EVOLUTION OF COSMOLOGICAL PERTURBATIONS

MATTER LOOPS IN GRAVITY

- RECALL: UNDER $x^\mu \to x^\mu + \xi^\mu(x)$ THE METRIC TRANSFORMS AS $g_{\mu\nu} \to g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$.

THEN THE 1PI EFFECTIVE ACTION

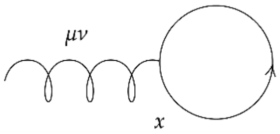
$$\Gamma[g_{\mu\nu}, \delta g_{\mu\nu}, \xi_\nu] = S_{\rm HE}[g_{\mu\nu}] + \frac{1}{2}\int d^D x \sqrt{-g}\, T^{\mu\nu}\delta g_{\mu\nu} + \frac{1}{2\kappa^2}\int d^D x d^D x' \sqrt{-g}\sqrt{-g'}\,\delta g_{\mu\nu} L^{\mu\nu\varrho\sigma}(x)\delta^D(x-x')\delta g_{\varrho\sigma}(x')$$

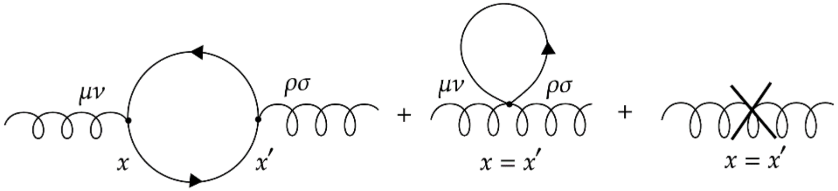
$$-\frac{1}{2}\int d^D x d^D x' \sqrt{-g}\sqrt{-g'}\,\delta g_{\mu\nu}(x)\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right](x;x')\delta g_{\varrho\sigma}(x') + O(\delta g^3)$$

WHERE $L^{\mu\nu\varrho\sigma}(x)$ IS LICHNEROWICZ (WAVE) OPERATOR, $g_{\mu\nu}$ IS THE BACKGROUND METRIC, WHICH ON-SHELL REDUCES TO: $g_{\mu\nu} \to \bar{g}_{\mu\nu}(x) = \langle \hat{g}_{\mu\nu}(x)\rangle$, AND $\delta g_{\mu\nu}$ IS A PERTUBATION. $T^{\mu\nu}$, $\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right]$ ARE THE GRAVITON 1- AND 2-POINT VERTEX FUNCTIONS, ALSO KNOWN AS THE ENERGY-MOMENTUM TENSOR AND SELF-ENERGY:

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g(x)}}\left\langle \frac{\delta S_{\rm m}}{\delta g_{\mu\nu}(x)}\right\rangle, \qquad -i\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right](x;x') = \frac{\kappa^2}{\sqrt{-g(x)}\sqrt{-g(x')}}\left\langle \frac{i\delta S_{\rm m}}{\delta g_{\mu\nu}(x)}\frac{i\delta S_{\rm m}}{\delta g_{\varrho\sigma}(x')} + \frac{i\delta^2 S_{\rm m}}{\delta g_{\mu\nu}(x)\delta g_{\varrho\sigma}(x')}\right\rangle$$

- WHEN APPROXIMATED AT THE 1-LOOP LEVEL, CONTRIBUTE AS κ^0 AND κ^2, RESPECTIVELY:

$T^{\mu\nu}(x) =$ 

$-i\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right](x;x') =$ 

SEMICLASSICAL THEORY OF GRAVITATIONAL PERTURBATIONS (SCGP)

SCGP: SEMICLASSICAL THEORY OF GRAVITATIONAL PERTURBATIONS

SEMICLASSICAL GRAVITY (SCG):

$$G_{\mu\nu}(g) = 8\pi G\big(T_{\mu\nu}(g,\psi_i) + \langle\delta\hat{T}_{\mu\nu}(g,\psi_i)\rangle\big), \qquad g_{\mu\nu} \equiv \langle\hat{g}_{\mu\nu}(g,\psi_i)\rangle = g^{cl}_{\mu\nu} + \delta g^{q}_{\mu\nu},$$

AND SEMICLASSICAL THEORY OF METRIC PERTURBATIONS FOR THE GRAVITON 1- AND 2-POINT FUNCTIONS (NB: no graviton loops):

$$L^{\mu\nu\varrho\sigma}(x)\delta g_{\varrho\sigma}(x') - \kappa^2 \int d^D x' \sqrt{-g'}\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right](x;x')\delta g_{\varrho\sigma}(x') + O(\delta g^2) = 0,$$

$$L^{\mu\nu\varrho\sigma}(x) i\left[{}_{\rho\sigma}\Delta_{\alpha\beta}\right](x;y) - \kappa^2 \int d^D x' \sqrt{-g'}\left[{}^{\mu\nu}\Sigma^{\varrho\sigma}\right](x;x') i\left[{}_{\rho\sigma}\Delta_{\alpha\beta}\right](x;y) + \,..= \frac{ih\delta^4(x-y)}{\sqrt{-g}}.$$

SINCE MOST OF THE EARLY UNIVERSE PROCESSES ARE QUANTUM (OR STATISTICAL) IN ORIGIN, THEY PRODUCE QUANTUM AMPLIFIED (CLASSICAL STOCHASTIC) PERTURBATIONS, THE SECOND EQUTATION IS MORE SUITABLE FOR MODELING THEIR DYNAMICS.

SCGP IN MINKOWSKI

- MASSIVE NON-MINIMALLY COUPLED SCALARS ON MINKOWSKI INDUCE CORRECTIONS TO GRAVITY (GRAVITATIONAL POTENTIALS AND GRAVITONS)

Jimu, Prokopec, JHEP (2025)

- THE DRESSED GRAVITON PROPAGATOR ALLOWS FOR ON-SHELL RENORMALIZATION SHEME ON LARGE DISTANCES (SMALL MOMENTA). THIS MEANS THAT THE LOOP CORRECTIONS CAN BE SUBSUMED TO THE FINITE RENORMALIZATION OF THE NEWTON CONSTANT, SUCH THAT WITH THE LOOP CORRECTIONS INCLUDED, THE OBSERVED NEWTON CONSTANT IS THAT OF THE THREE LEVEL THEORY. THIS IS A GRAVITATIONAL ANALOG OF THE APPELQUIST-CARAZONE THEOREM (LOOP EFFECTS OF HIGHLY MASSIVE PARTICLES CAN BE SUBSUMED IN THE SUITABLE RENORMALIZATION OF THE TREE-LEVEL PARAMETERS):

$$\frac{1}{16\pi G_0} = \frac{1}{16\pi G} + \frac{\Omega m^2 \mu^{D-4}}{2(D-4)}\left(\xi - \frac{1}{6}\right) \qquad c_3 = c_{3\mathrm{div}} + c_{3\mathrm{f}} = \frac{\Omega m^2}{2}\left(\xi - \frac{1}{6}\right)\left(\frac{\mu^{D-4}}{D-4} + \Gamma_E\right)$$

► GRAVITATIONAL SCALARS RESPOND TO A POINT MASS AS:

$$\Phi(r) = -\frac{GM}{r}\left\{1 + \frac{Gm^2}{\pi}\left[\left(\frac{1}{10} + 2\xi^2 - \frac{2\xi}{3}\right)\frac{K_1(2mr)}{mr} - \left(\frac{1}{15} + \frac{2\xi}{3}\right)\left(K_0(2mr) - \mathrm{Ki}_2(2mr)\right)\right.\right.$$
$$\left.\left. + \frac{7}{60}\left(\mathrm{Ki}_2(2mr) - \mathrm{Ki}_4(2mr)\right)\right]\right\},$$
$$\Psi(r) = -\frac{GM}{r}\left\{1 - \frac{Gm^2}{\pi}\left[\left(\frac{1}{30} + 2\xi^2 - \frac{2\xi}{3}\right)\frac{K_1(2mr)}{mr} - \left(-\frac{1}{5} + \frac{2\xi}{3}\right)\left(K_0(2mr) - \mathrm{Ki}_2(2mr)\right)\right.\right.$$
$$\left.\left. - \frac{1}{60}\left(\mathrm{Ki}_2(2mr) - \mathrm{Ki}_4(2mr)\right)\right]\right\}.$$

► HERE $Ki_n(z)$ DENOTES THE BICKLEY FUNCTION: $$Ki_n(z) = \int_0^\infty dx\, \frac{e^{-z\cosh(x)}}{[\cosh(x)]^n}$$

° $Ki_n(z)$ ARE RELATED TO MACDONALD FUNCTIONS (BESSEL FUNCTIONS OF THE 2nd KIND) AS:

$$Ki_0(z) = K_0(z), \qquad Ki_n(z) = \int_0^\infty dx Ki_{n-1}(x)$$

SCGP IN MINKOWSKI II

► GRAVITATIONAL SCALARS RESPOND TO A POINT MASS:

- AT ASYMPTOTICALLY LARGE DISTANCES $(mr \gg 1)$:

Jimu, Prokopec, JHEP (2025)

$$\Phi(r)|_{r\to\infty} = -\frac{GM}{r}\left\{1 + G\sqrt{\frac{m}{\pi r^3}}e^{-2mr}\left[\left(\xi - \frac{1}{4}\right)^2 + \frac{3}{16mr}\left(\xi - \frac{1}{4}\right)\left(\xi + \frac{13}{12}\right)\right]\right\}$$

$$\Psi(r)|_{r\to\infty} = -\frac{GM}{r}\left\{1 - G\sqrt{\frac{m}{\pi r^3}}e^{-2mr}\left[\left(\xi - \frac{1}{4}\right)^2 + \frac{3}{16mr}\left(\xi - \frac{1}{4}\right)\left(\xi + \frac{13}{12}\right)\right]\right\}$$

◦ QUALITATIVELY THE SAME ASYMPTOTIC BEHAVIOUR AS IN COULOMB POTENTIAL IN QED (UHLENBECK POTENTIAL)

- AT SMALL DISTANCES $(mr \ll 1)$:

$$\Phi(r)|_{r\to 0} = -\frac{GM}{r}\left\{1 + \frac{G}{\pi r^2}\left(\frac{1}{20} + \xi^2 - \frac{\xi}{3}\right) + \frac{Gm^2}{\pi}\left[\left(\frac{1}{6} + 2\xi^2\right)\left(\ln(mr) + \gamma_E\right) + \frac{1}{18} - \xi^2 + \xi\right]\right\},$$

$$\Psi(r)|_{r\to 0} = -\frac{GM}{r}\left\{1 - \frac{G}{\pi r^2}\left(\frac{1}{60} + \xi^2 - \frac{\xi}{3}\right) - \frac{Gm^2}{\pi}\left[\left(-\frac{1}{6} + 2\xi^2\right)\left(\ln(mr) + \gamma_E\right) - \frac{2}{9} - \xi^2 + \xi\right]\right\}$$

◦ ADDITIONAL ~ ln(mr) WHEN COMPARED WITH THE UHLENBECK POTENTIAL.

► NON-MINIMAL COUPLING INDUCES A GRAVITATIONAL SLIP: $\Sigma = \Phi - \Psi$

$$\frac{\Sigma}{\Phi_0} = \frac{2G}{\pi r^2}\left(\xi^2 - \frac{\xi}{3} + \frac{1}{30}\right) + \frac{2Gm^2}{\pi}\left[2\xi^2\left(\ln(mr) + \gamma_E\right) - \xi^2 + \xi - \frac{1}{12}\right]$$

◦ THE ABSENCE OF GRAVITATIONAL SLIP IN SOLAR SYSTEM EXPERIEMNTS $(\Sigma < 2\times 10^{-5}\Phi)$ PROVIDES UNREMARKABLE UPPER BOUND ON THE NONMINIMAL COUPLING: $|\xi| \lesssim 5\times 10^{43}/\sqrt{N}$

SCGP IN INFLATION

- ONLY RESULTS FROM THE GRAVITON SELF-ENERGY INDUCED BY MMCS ARE KNOWN.

► GRAVITON 1-POINT FUNCTION DOES NOT GET SIGNIFICANTLY AFFECTED, HOWEVER THE CORRESPONDING WEYL CURVATURE TENSOR GROWS AS $\propto \hbar G H^2 \log(a)$

► GRAVITATIONAL SCALARS RESPOND SECULARLY TO A POINT MASS:

Park, Prokopec, Woodard, PRD, 1403.0896 (2014) 1510.03352

$$\phi_{\rm dS}(x) = -\frac{GM}{ar}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\left(\frac{Har}{c}\right)\right] + \mathcal{O}\left(G^2, \frac{1}{a^3}\right)\right\} \qquad (56)$$

$$\psi_{\rm dS}(x) = -\frac{GM}{ar}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\left(\frac{Har}{c}\right) + \frac{2}{3}\frac{Har}{c}\right] + \mathcal{O}\left(G^2, \frac{1}{a^3}\right)\right\}.$$

► GRAVITONS GENERATE A FORCE OF GRAVITY IN DE SITTER [GAUGE DEPENDENT]:

$$\kappa^2 \Psi_1(\eta, r) \longrightarrow \frac{2GM}{ar}\left\{-\frac{4GH^2\ln^3(a)}{\pi} + \frac{12GH^2\ln(a)\ln(Hr)}{\pi}\right\}$$

Tan, Tsamis, Woodard, PRD, 2206.11467 [gr-qc]

- PHYSICAL RAMIFICATIONS OF THESE SECULAR EFFECTS ARE NOW YET WELL UNDERSTOOD!
- THE EFFECTS FROM MMCS GROW LINERALY IN TIME, EVENTUALLY GROW SO BIG SO THAT $\hbar G H^2 \log(a) > O(1)$, INVALIDATING PERTURBATION THEORY $\Rightarrow$ RESUMMATION NEEDED!

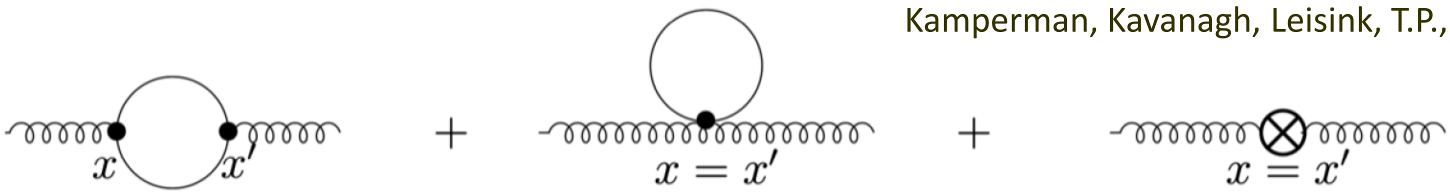
$\Rightarrow$ SEE DRAZEN GLAVAN'S TALK ON SECULAR EFFECTS ON DYNAMICAL GRAVITONS IN RADIATION!

FROB, GLAVAN, MEDA, SAWICKI (2025)

SCGP IN INFLATION II

- THE GRAVITON SELF-ENERGY FROM A MASSIVE NONMINIMALLY COUPLED SCALAR

Kamperman, Kavanagh, Leisink, T.P., in progress (2025)



▶ DIMENSIONALLY REGULATED RENORMALIZED RESULT

Riley Kavanagh, T.P. (UU master thesis 2023)

$$
\begin{aligned}
& -i\left[{}_{\mu\nu}\Sigma^{\rm Ren}_{\rho\sigma}\right]^{\rm local}(x;x') = -\frac{i\hbar\kappa^2}{(4\pi)^4}\Bigg\{ g_{\mu\nu}g'_{\rho\sigma}\Box\Box'\left[\frac{1}{6}\right] \\
&+ H^2\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right)\Bigg[-\frac{5}{24} + (1+6\xi)\Big(\tfrac{1}{24} + \xi\left[\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12 \pm \nu)\right]\Big) \\
&\quad + \frac{1}{3}(1+6\xi)^2 + \frac{m^2}{H^2}\Big(-\tfrac{1}{24} - \tfrac{1}{24}(1+6\xi) - \tfrac12\xi\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right)\Big)\Bigg] \\
&+ H^2 g_{\mu\nu}g'_{\rho\sigma}\Box\Bigg[-\frac{221}{120} - (1+6\xi)\Big(\tfrac38 + \xi\left[\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right]\Big) - \tfrac{1}{12}(1+6\xi)^2 \\
&\quad + \frac{m^2}{H^2}\Big(\tfrac12\xi\left[\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right] - \tfrac{1}{12} + \tfrac{1}{12}(1+6\xi)\Big)\Bigg] \\
&+ H^2\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box\Bigg[-\frac{1}{12} + \frac18(1+6\xi) - \frac12\xi\Big(\tfrac{m^2}{H^2} - 2(1+6\xi)\Big)\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right)\Bigg] \\
&+ H^2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Bigg[-\frac{22}{120} + \frac14(1+6\xi) - \xi\Big(\tfrac{m^2}{H^2} - 2(1+6\xi)\Big)\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right)\Bigg] \\
&+ H^4 g_{\mu\nu}g'_{\rho\sigma}\Bigg[\frac{m^2}{H^2}\Big(-\tfrac18 - \tfrac12(1+6\xi)\left[\psi(1) - \tfrac34 - \tfrac12\psi(\tfrac12\pm\nu)\right]\Big) \\
&\quad + \frac14\left(\frac{m^2}{H^2}\right)^2\left[\psi(1) - \tfrac12\psi(\tfrac12\pm\nu)\right] - \frac{47}{80} + (-\tfrac{79}{24} - 6\xi)(1+6\xi) - \frac{1}{16}(1+6\xi)^2\Bigg] \\
&+ H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg[\frac14 - \frac34(1+6\xi) - \frac14\frac{m^2}{H^2} + (\tfrac14 - \nu^2)\Big(\left(\psi(1) - \tfrac74 - \tfrac12\psi(\tfrac12\pm\nu)\right) \\
&\qquad + \frac12\frac{m^2}{H^2}\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right) - 3\xi\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right)\Big) \\
&\qquad - \frac12\left(\tfrac14 - \nu^2\right)^2\left(\psi(1) - \tfrac12 - \tfrac12\psi(\tfrac12\pm\nu)\right)\Bigg]\Bigg\}\frac{\delta^4(x-x')}{\sqrt{-g}}, \qquad (4.27)
\end{aligned}
$$

SCGP IN INFLATION III

► DIMENSIONALLY REGULATED RENORMALIZED RESULT (cont)

Riley Kavanagh, T.P. (UU master thesis 2023)

$$-i\left[{}_{\mu\nu}\Sigma^{\rm Ren}_{\rho\sigma}\right]^{\text{non-local}}(x;x') = \frac{\hbar\kappa^2 H^4}{4(4\pi)^4}\times$$

$$\Big\{\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma \times \Big[\left(\tfrac{1}{90}+\tfrac{1}{18}(1+6\xi)^2\right)g(y) - \left(\tfrac14-\nu^2\right)^2\ln\frac{y}{4}$$

$$+\left(\tfrac14-\nu^2\right)\left(\left(\tfrac13-\tfrac23(1+6\xi)\right)\frac{4}{y}-\tfrac43\ln\frac{y}{4}\right)+2\xi^2\left(\tfrac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\right)\Big[\left(\tfrac{1}{180}-\tfrac{1}{18}(1+6\xi)^2\right)g(y)+\tfrac12\left(\tfrac14-\nu^2\right)^2\ln\frac{y}{4}$$

$$+\left(\tfrac14-\nu^2\right)\left(\left(-\tfrac13+\tfrac12(1+6\xi)\right)\frac{4}{y}+\tfrac23\ln\frac{y}{4}\right)-2\xi^2\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+g_{\mu\nu}g'_{\rho\sigma}\Box\Box'\Big[\left(-\tfrac{1}{180}+\tfrac{1}{18}(1+6\xi)^2\right)g(y)-\tfrac14\left(\tfrac14-\nu^2\right)^2\ln\frac{y}{4}$$

$$+\left(\tfrac14-\nu^2\right)\left(\tfrac{7}{12}-\tfrac23(1+6\xi)\frac{4}{y}-\tfrac13\ln\frac{y}{4}\right)+2\xi^2\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\Box\Big[\left(\tfrac{1}{30}\right)g(y)\Big]+\left[{}_{(\mu}g_{|\rho|}\right]\left[{}_{\nu)}g_{\sigma}\right]\Box\Box\Big[\left(\tfrac{1}{60}\right)g(y)\Big]$$

$$+H^2\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right)\Big[\left(-\tfrac{1}{30}-\tfrac16\frac{m^2}{H^2}+\tfrac13(1+6\xi)-\tfrac16(1+6\xi)^2\right)g(y)$$

$$-\frac{4}{y}-2\ln\frac{y}{4}+\left(\tfrac14-\nu^2\right)\Big(\Big(2\big[\psi(1)-\tfrac{1}{12}-\tfrac12\psi(\tfrac12\pm\nu)\big]-\tfrac13(1+6\xi)+\frac{m^2}{H^2}\Big)\frac{4}{y}$$

$$+4\big[\psi(1)-\tfrac34-\tfrac12\psi(\tfrac12\pm\nu)\big]\ln\frac{y}{4}-\frac{\ln y/4}{y/4}-\left(\ln\frac{y}{4}\right)^2\Big)$$

$$+\left(\tfrac14-\nu^2\right)^2\Big(\big[\psi(1)-\tfrac34-\tfrac12\psi(\tfrac12\pm\nu)\big]\frac{4}{y}+2\big[\psi(1)+\tfrac14-\tfrac12\psi(\tfrac12\pm\nu)\big]\ln\frac{y}{4}$$

$$-\frac12\frac{\ln y/4}{y/4}-\frac12\left(\ln\frac{y}{4}\right)^2\Big)+\left(-\frac{m^2}{H^2}\xi+6\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+H^2g_{\mu\nu}g'_{\rho\sigma}\Box\Big[\left(\tfrac{13}{60}-\tfrac23(1+6\xi)+\tfrac13(1+6\xi)^2+\tfrac16\frac{m^2}{H^2}\right)g(y)+\tfrac12\frac{4}{y}-\ln\frac{y}{4}$$

$$+\left(\tfrac14-\nu^2\right)\Big(\Big(-\big[\psi(1)-\tfrac14-\tfrac12\psi(\tfrac12\pm\nu)\big]+2(1+6\xi)-\frac{m^2}{H^2}\Big)\frac{4}{y}$$

$$-2\big[\psi(1)-\tfrac34-\tfrac12\psi(\tfrac12\pm\nu)\big]\ln\frac{y}{4}+\frac12\frac{\ln y/4}{y/4}+\frac12\left(\ln\frac{y}{4}\right)^2\Big)$$

$$+\left(\tfrac14-\nu^2\right)^2\Big(-\tfrac12\big[\psi(1)-\tfrac54-\tfrac12\psi(\tfrac12\pm\nu)\big]\frac{4}{y}-2\big[\psi(1)+\tfrac14-\tfrac12\psi(\tfrac12\pm\nu)\big]\ln\frac{y}{4}$$

$$-\frac14\frac{\ln y/4}{y/4}-\frac14\left(\ln\frac{y}{4}\right)^2\Big)+\left(2\frac{m^2}{H^2}\xi-12\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\Big[-\tfrac{1}{15}g(y)-2\frac{4}{y}$$

$$+\left(\tfrac14-\nu^2\right)\left(-\tfrac13 g(y)+4\big[\psi(1)-\tfrac{17}{12}-\tfrac12\psi(\tfrac12\pm\nu)\big]\frac{4}{y}-2\frac{\ln y/4}{y/4}\right)$$

$$+\left(\tfrac14-\nu^2\right)^2\left(2\big[\psi(1)-\tfrac14-\tfrac12\psi(\tfrac12\pm\nu)\big]\frac{4}{y}-4\frac{\ln y/4}{y}\right)\Big]$$

$$+H^2\left[{}_{(\mu}g_{|\rho|}\right]\left[{}_{\nu)}g_\sigma\right]\Box\Big[-\tfrac{1}{30}g(y)-\tfrac16\left(\tfrac14-\nu^2\right)g(y)\Big]$$

$$+H^4g_{\mu\nu}g'_{\rho\sigma}\Big[\Big(-\tfrac{1}{10}+\tfrac14\left(\frac{m^2}{H^2}\right)^2+\frac{m^2}{H^2}(1+6\xi)\Big)g(y)+\left(\tfrac14-\nu^2\right)\left(1+2\frac{m^2}{H^2}-2(1+6\xi)\right)\frac{4}{y}$$

$$-\tfrac14\left(\tfrac14-\nu^2\right)^2\frac{4}{y}+\left(\frac12\left(\frac{m^2}{H^2}\right)^2-6\frac{m^2}{H^2}\xi+18\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Big]$$

$$+H^4\left[{}_{(\mu}g_{|\rho|}\right]\left[{}_{\nu)}g_\sigma\right]\Big[\frac12\left(\frac{m^2}{H^2}\right)^2+\frac{m^2}{H^2}\left(1-2(1+6\xi)\right)-2(1+6\xi)+2(1+6\xi)^2\Big]g(y)$$

$$+\left(2\delta^\alpha_{(\mu}\delta^\beta_{\nu)}\delta^\gamma_{(\rho}\delta^\delta_{\sigma)}-\left(g_{\mu\nu}g^{\alpha\beta}\delta^\gamma_{(\rho}\delta^\delta_{\sigma)}+g'_{\rho\sigma}g'^{\gamma\delta}\delta^\alpha_{\mu}\delta^\beta_{\nu}\right)+\frac12 g_{\mu\nu}g'_{\rho\sigma}g^{\alpha\beta}g'^{\gamma\delta}\right)$$

$$\times\Big[2\partial_\alpha\partial'_\gamma\frac{4}{y}\partial_\beta\partial'_\delta f_2(y)+2\left(\tfrac14-\nu^2\right)\partial_\alpha\partial'_\gamma\ln(y/4)\partial_\beta\partial'_\delta f_1(y)+\partial_\alpha\partial'_\gamma f_1(y)\partial_\beta\partial'_\delta f_1(y)\Big]$$

$$+\Big[(m^2-12H^2\xi)g_{\mu\nu}g'_{\rho\sigma}-\xi(g_{\mu\nu}\mathcal{P}'_{\rho\sigma}+g'_{\rho\sigma}\mathcal{P}_{\mu\nu})\Big]\left(2\partial^\alpha\frac{4}{y}\partial_\alpha f_1(y)+\partial^\alpha f_1(y)\partial_\alpha f_1(y)\right)$$

$$+\Big[-(m^2-12H^2\xi)g_{\mu\nu}+2\xi\mathcal{P}_{\mu\nu}\Big]\left(2\partial'_{(\rho}\frac{4}{y}\partial'_{\sigma)}f_1(y)+\partial'_\rho f_1(y)\partial'_\sigma f_1(y)\right)$$

$$+\Big[-(m^2-12H^2\xi)g'_{\rho\sigma}+2\xi\mathcal{P}'_{\rho\sigma}\Big]\left(2\partial_{(\mu}\frac{4}{y}\partial_{\nu)}f_1(y)+\partial_\mu f_1(y)\partial_\nu f_1(y)\right)\Big\}, \qquad (4.28)$$

Kamperman, Leisink, T.P., in progress (2025)

► WE ARE CHECKING THIS RESULT, TRYING TO PUT IT INTO A MOSTLY TRANSVERSE FORM, UP TO A LOCAL CONTRIBUTION REMOVABLE BY A FINITE COSMOLOGICAL CONST. COUNTERTERM.

SCGP IN INFLATION IV: RENORMALIZATION

- FOUR COUNTERTERMS ARE NEEDED:

$$\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^D x''\sqrt{-g}\Big\{c_1 C^{\alpha\gamma\beta\delta}C_{\alpha\gamma\beta\delta}+c_2R^2+c_3m^2R+c_4m^4\Big\}$$

$$=\sqrt{-g(x)}\sqrt{-g(x')}\bigg\{c_1\frac{2(D-3)}{(D-2)}\mathcal{F}_{\mu\nu\rho\sigma}(x;x')+c_2\mathcal{G}_{\mu\nu\rho\sigma}(x;x')$$

$$+c_3\mathcal{P}_{\mu\nu\rho\sigma}(x;x')+c_4\frac{1}{4}\Big(g_{\mu\nu}g_{\rho\sigma}+2\,{}_{[\mu}g_{\rho]}\,{}_{[\nu}g_{\sigma]}\Big)\bigg\}\frac{\delta^D(x-x')}{\sqrt{-g}}$$

$$c_1=\frac{\hbar\kappa^2\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{(D-2)}{16(D+1)(D-1)(D-3)^2}\frac{\mu^{D-4}}{(D-4)}+c_1^f$$

$$c_2=\frac{\hbar\kappa^2\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{\mu^{D-4}}{2(D-3)(D-4)}\left(\frac{1}{4}\frac{(D-2)}{(D-1)}+\xi\right)^2+c_2^f$$

$$c_3=\cancel{-}\frac{\hbar\kappa^2\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{\mu^{D-4}}{(D-3)(D-4)}\left(\frac{1}{4}\frac{(D-2)}{(D-1)}+\xi\right)+c_3^f$$

$$c_4=\frac{\hbar\kappa^2\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{\mu^{D-4}}{2(D-3)(D-4)}+c_4^f\,.$$

NB: THE LAST TERM IS NON-TRANSVERSE, BUT IT CAN BE REMOVED BY A FINITE COSMOLOGICAL CONSTANT COUNTERTERM.

NB2: THE SAME COUNTERTERM REMOVES THE ONE-LOOP CONTRIBUTION TO THE ENERGY-MOMENTUM TENSOR.

THIS MEANS THAT GRAVITATIONAL PERTURBATIONS REMAIN PERTURBATIONS IN THE SENSE THAT THEY DO NOT ACQUIRE AN EXPECTATION VALUE WITH TIME.

SCGP IN INFLATION V: TRANSVERSALITY

Kamperman, Leisink, T.P., in progress (2025)

► WE ARE CHECKING THIS RESULT, TRYING TO PUT IT INTO A MOSTLY TRANSVERSE FORM, UP TO A LOCAL CONTRIBUTION REMOVABLE BY A FINITE COSMOLOGICAL CONST. COUNTERTERM.

- $P_{\mu\nu}(x)P_{\rho\sigma}(x')$:
- $C_{\mu\nu\rho\sigma\alpha\beta}(x)C_{\gamma\delta\varphi\tau\omega\psi}(x') \times [^{\rho}g^{\varphi}](x;x')$ ...

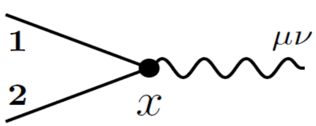
► WHERE: $P_{\mu\nu}(x)$ IS TRANSVERSE PROJECTOR ON DE SITTER:

$$\mathcal{P}_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu}\Big(\Box + H^2(D-1)\Big)$$

► $C_{\mu\nu\rho\sigma\alpha\beta}(x)$ A METRIC DERIVATIVE OF THE WEYL TENSOR.

► WHY IS THIS PROBLEM IMPORTANT?

• AS OPPOSED TO MMCS, MASSIVE SCALAR CONTAINS NON-DERIVATIVE VERTICES, MAKING THE GRAVITON SENSITIVE TO IR SCALAR PERTUBATIONS, WHICH CAN BE HIGHLY ENHANCED.



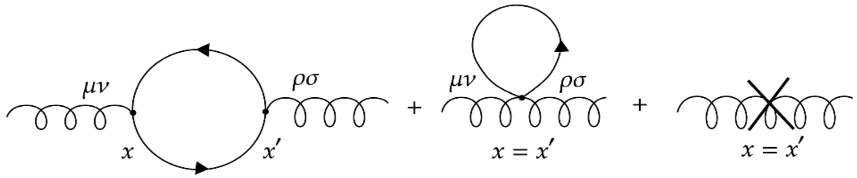
$$= \; -i\kappa a_x^{D-2}\Big[-\partial_1^\mu \partial_2^\nu + \tfrac{1}{2}\eta^{\mu\nu}\big(\partial_1 \cdot \partial_2 + a_x^2 m^2\big)\Big]$$

- WE CAN RIGOROUSLY STUDY THE EFFECT (DECAY) OF (LIGHT & HEAVY) MASSIVE SCALARS ON GRAVITONS (`COSMOLOGICAL COLLIDER PHYSICS' , see Masahide Yamaguchi's talk)
- NEGATIVE NONMINIMAL COUPLING ACTS AS A TACHYONIC MASS, AND CAN FURTHER ENHANCE THE IR SCALAR FLUCTUATIONS (AT THE MOMENT WE ARE STUDYING $m^2 + \xi R > 0$).

SCGP IN RADIATION ERA

Liu, T.P., in progress (2025)

- THE GRAVITON SELF-ENERGY FROM MASSIVE NONMINIMALLY COUPLED SCALAR



- ADIABATIC APPROXIMATION, 2 REGIMES: $T^2, m^2 \gg H^2$ OR $T^2 \gg H^2 \gg m^2$.

NB: FOR MASSLESS SCALAR COUPLE CONFORMALLY IN RADIATION, THEREFORE THERMAL LOOP EFFECTS IN THE MASSLESS LIMIT CAN BE TREATED EXACTLY.

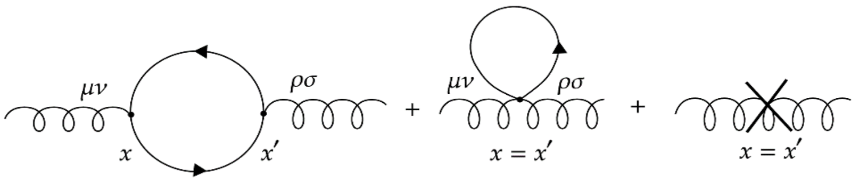
- WE HAVE NOW COMPUTED THE GRAVITON SELF-ENERGY AT LEADING ORDER IN ADIABATIC APPROXIMATION AND PUT IT (IN PART) IN A TRANSVERSE FORM.

- PROPAGATOR $i\Delta(x;x') = i\Delta_{\rm v}(x;x') + i\Delta_{\rm th}(x;x')$
- SELF-ENERGY $i\Sigma(x;x') = i\Sigma_{\rm v}(x;x') + i\Sigma_{v-{\rm th}}(x;x') + i\Sigma_{\rm th}(x;x')$
- THE VACUUM PART $i\Sigma_{\rm v}(x;x')$ IS FULLY REGULARIZED AND RENORMALIZED & IN A MANIFESTLY TRANSVERSE FORM
- THE THERMAL PART $i\Sigma_{\rm th}(x;x')$ (ONLY FROM THE 3PT DIAGRAM) IS BY ITSELF TRANSVERSE.
- THE VACUUM-THERMAL PART $i\Sigma_{v-{\rm th}}(x;x')$ IS TRICKY. WE HAVE NOT YET ESTABLISHED THAT ITS LOCAL PART SATISFIES A WARD IDENTITY (work in progress).

SCGP IN RADIATION ERA II

Liu, T.P., in progress (2025)

- SELF-ENERGY FROM MASSIVE NONMINIMALLY COUPLED SCALAR, THERMAL PART



$$
\begin{aligned}
-i[{}_{\mu\nu}\Sigma^{\rm 3pt,th}_{\rho\sigma}]^{\rm I}_{\xi=0} = \frac{\hbar\kappa^2}{4}\sum_{t=1}^{4}\Big\{ & \overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}\Big[\frac{J^t_{40}+2J^t_{22}+J^t_{04}}{4}+J^t_{20}\Big(a^2m^2+\frac{k^2}{4}\Big) \\
& \qquad + J^t_{02}\Big(a^2m^2-\frac{3k^2}{4}\Big)+J^t_{00}\Big(a^4m^4+\frac{k^4}{16}\Big)\Big] \\
& + \overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu}\Big[\frac{J^t_{40}-2J^t_{22}+J^t_{04}}{2}\Big] \\
& + \frac{\overline{k}_\mu\overline{k}_\nu\overline{\eta}_{\rho\sigma}+\overline{\eta}_{\mu\nu}\overline{k}_\rho\overline{k}_\sigma}{k^2}\Big[\frac{J^t_{40}-5J^t_{04}}{4}+J^t_{20}\Big(\frac{a^2m^2}{2}-\frac{k^2}{8}\Big) \\
& \qquad + J^t_{02}\Big(-\frac{3a^2m^2}{2}+\frac{5k^2}{8}\Big)+J^t_{00}\Big(a^2m^2-\frac{k^2}{4}\Big)\frac{k^2}{4}\Big] \\
& + \frac{\overline{k}_{(\mu}\overline{\eta}_{\nu)(\rho}\overline{k}_{\sigma)}}{k^2}\Big[-J^t_{40}+6J^t_{22}-5J^t_{04}\Big] \\
& + \frac{\overline{k}_\mu\overline{k}_\nu\overline{k}_\rho\overline{k}_\sigma}{k^4}\Big[\frac{3J^t_{40}-30J^t_{22}+35J^t_{04}}{4}+k^2\frac{J^t_{20}-3J^t_{02}}{2}+\frac{k^4}{8}J^t_{00}\Big] \\
& + \delta^0_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0_{\sigma)}\Big[2(J^t_{40}-J^t_{22})+2\Big(a^2m^2+\frac{k^2}{4}\Big)(J^t_{20}-J^t_{02})\Big] \\
& + \frac{\overline{k}_{(\mu}\delta^0_{\nu)}\delta^0_{(\rho}\overline{k}_{\sigma)}}{k^2}\Big[2(-J^t_{40}+3J^t_{22})+J^t_{20}\Big(-2a^2m^2+\frac{k^2}{2}\Big)+J^t_{02}\Big(6a^2m^2-\frac{5k^2}{2}\Big) \\
& \qquad + k^2\Big(a^2m^2+\frac{k^2}{4}\Big)J^t_{00}\Big] \\
& + \delta^0_\mu\delta^0_\nu\delta^0_\rho\delta^0_\sigma\Big[J^t_{40}+2a^2m^2J^t_{20}-\frac{k^2}{2}J^t_{02}+J^t_{00}\Big(a^4m^4+\frac{k^4}{16}\Big)\Big] \\
& + (\delta^0_\mu\delta^0_\nu\overline{\eta}_{\rho\sigma}+\overline{\eta}_{\mu\nu}\delta^0_\rho\delta^0_\sigma)\Big[\frac{J^t_{40}-J^t_{22}}{2}+\Big(\frac{a^2m^2}{2}+\frac{3k^2}{8}\Big)(J^t_{20}-J^t_{02})+J^t_{00}\Big(\frac{a^2m^2k^2}{2}\Big)\Big] \\
& + \frac{\delta^0_\mu\delta^0_\nu\overline{k}_\rho\overline{k}_\sigma+\overline{k}_\mu\overline{k}_\nu\delta^0_\rho\delta^0_\sigma}{k^2}\Big[\frac{-J^t_{40}+3J^t_{22}}{2}+J^t_{20}\Big(-\frac{a^2m^2}{2}-\frac{k^2}{8}\Big) \\
& \qquad + J^t_{02}\Big(\frac{3a^2m^2}{2}-\frac{3k^2}{8}\Big)+J^t_{00}\Big(-a^2m^2+\frac{k^2}{4}\Big)\frac{k^2}{4}\Big]\Big\}, \qquad (87)
\end{aligned}
$$

$$
\begin{aligned}
-i[{}_{\mu\nu}\Sigma^{\rm 3pt,th}_{\rho\sigma}]^{\rm II}_{\xi=0}(\eta,\eta';\mathbf{k}) = \frac{\hbar\kappa^2}{4}\sum_{t=1}^{4}s^t_+s^t_- & \\
\times\Big\{ & \overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}\Big[L^t_{02}+\Big(a^2m^2-\frac{k^2}{4}\Big)L^t_{00}\Big] \\
& + \frac{\overline{k}_\mu\overline{k}_\nu\overline{\eta}_{\rho\sigma}+\overline{\eta}_{\mu\nu}\overline{k}_\rho\overline{k}_\sigma}{k^2}\Big[\frac{L^t_{20}-3L^t_{02}}{2}+\frac{k^2}{4}L^t_{00}\Big] \\
& + \delta^0_\mu\delta^0_\nu\delta^0_\rho\delta^0_\sigma\Big[-L^t_{20}+\Big(-a^2m^2+\frac{k^2}{4}\Big)L^t_{00}\Big] \\
& + (\delta^0_\mu\delta^0_\nu\overline{\eta}_{\rho\sigma}+\overline{\eta}_{\mu\nu}\delta^0_\rho\delta^0_\sigma)\Big[\frac{-L^t_{20}+L^t_{02}}{2}\Big] \\
& + \delta^0_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0_{\sigma)}\Big[2(-L^t_{20}+L^t_{02})\Big] \\
& + \frac{\delta^0_\mu\delta^0_\nu\overline{k}_\rho\overline{k}_\sigma+\overline{k}_\mu\overline{k}_\nu\delta^0_\rho\delta^0_\sigma}{k^2}\Big[\frac{L^t_{20}-3L^t_{02}}{2}+\frac{k^2}{4}L^t_{00}\Big] \\
& + \frac{\overline{k}_{(\mu}\delta^0_{\nu)}\delta^0_{(\rho}\overline{k}_{\sigma)}}{k^2}\Big[2(L^t_{20}-3L^t_{02})+k^2L^t_{00}\Big]\Big\},
\end{aligned}
$$

NB: ONLY THE TRACE TENSOR STRUCTURE $\eta_{\mu(\rho}\eta_{\sigma)\nu}$ COUPLES TO THE DYNAMICAL GRAVITON.

- WE HAVE NOT BEEN ABLE TO PUT IT INTO A MANIFESTLY TRANSVERSE FORM. NOT YET.

- SIMPLE INTEGRALS:

$$
\begin{aligned}
J^t_{rs}(\eta,\eta';k^2) &= \frac{1}{4}\int\frac{d^3q}{(2\pi)^3}\frac{n_+n_-}{\omega_+\omega_-}\Big[q^r\frac{(\mathbf{k}\cdot\mathbf{q})^s}{k^s}\Big]\times e^{i(s^t_+\omega_++s^t_-\omega_-)\Delta\eta}, \\
& \qquad (r,s=0,2,4;\ t=1,2,3,4), \\
L^t_{rs}(\eta,\eta';k^2) &= \frac{1}{4}\int\frac{d^3q}{(2\pi)^3}n_+n_-\Big[q^r\frac{(\mathbf{k}\cdot\mathbf{q})^s}{k^s}\Big]\times e^{i(s^t_+\omega_++s^t_-\omega_-)\Delta\eta}, \\
N^t_{rs}(\eta,\eta';k^2) &= \frac{1}{4}\int\frac{d^3q}{(2\pi)^3}\frac{n_+n_-}{\omega_+}\Big[q^r\frac{(\mathbf{k}\cdot\mathbf{q})^s}{k^s}\Big]\times e^{i(s^t_+\omega_++s^t_-\omega_-)\Delta\eta} \\
M^t_{rs}(\eta,\eta';k^2) &= \frac{1}{4}\int\frac{d^3q}{(2\pi)^3}\frac{n_+n_-}{\omega_-}\Big[q^r\frac{(\mathbf{k}\cdot\mathbf{q})^s}{k^s}\Big]\times e^{i(s^t_+\omega_++s^t_-\omega_-)\Delta\eta}, \\
&= (-1)^sN^t_{rs}, \qquad (s=0,1,2,3;\ r=0,2,4).
\end{aligned}
$$

$$
\begin{aligned}
-i[{}_{\mu\nu}\Sigma^{\rm 3pt,th}_{\rho\sigma}]^{\rm III}_{\xi=0}(\eta,\eta';\mathbf{k}) = \frac{\hbar\kappa^2}{4}\sum_{t=1}^{4}s^t_-\Big\{ & \frac{k_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0_{\sigma)}+\delta^0_{(\mu}\overline{\eta}_{\nu)(\rho}k_{\sigma)}}{k}\Big[-4\,(N^t_{21}-N^t_{03})\Big] \\
& + \frac{\overline{\eta}_{\mu\nu}\overline{k}_{(\rho}\delta^0_{\sigma)}+\overline{k}_{(\mu}\delta^0_{\nu)}\overline{\eta}_{\rho\sigma}}{k}\Big[-2(N^t_{21}-N^t_{03})+k(N^t_{20}-N^t_{02})+N^t_{00}(2ka^2m^2)\Big] \\
& + \frac{\overline{k}_\mu\overline{k}_\nu\overline{k}_{(\rho}\delta^0_{\sigma)}+\overline{k}_{(\mu}\delta^0_{\nu)}\overline{k}_\rho\overline{k}_\sigma}{k^3}\Big[2\,(3N^t_{21}-5N^t_{03})+k\,(N^t_{20}-3N^t_{02}) \\
& \qquad + k^2N^t_{01}+\frac{k^3}{2}N^t_{00}\Big] \\
& + \frac{\delta^0_\mu\delta^0_\nu\overline{k}_{(\rho}\delta^0_{\sigma)}+\overline{k}_{(\mu}\delta^0_{\nu)}\delta^0_\rho\delta_\sigma}{k}\Big[-4N^t_{21}-2kN^t_{02}+N^t_{01}(-4a^2m^2+k^2)+\frac{k^3}{2}N^t_{00}\Big]\Big\},
\end{aligned}
$$

SCGP IN RADIATION AND MATTER ERA III

Liu, T.P., in progress (2025)

- NAÏVE ESTIMATE: $\hbar G T^2 \sim \frac{H}{M_P} \sim \frac{T^2}{M_P^2} \sim 10^{-6}$ AT PREHEATING $\sim 10^{-16}$ AT ELECTROWEAK SCALE.

- RADIATION ERA GRAVITON 1-POINT FUNCTION (TT gauge) & RETARDED SELF-ENERGY:

$h_{ij}(\eta, \boldsymbol{k}) = \epsilon_{ij}^{\alpha}(\boldsymbol{k}) h_\alpha(\eta, \boldsymbol{k}), h_\alpha(\eta, \boldsymbol{k}) = A_\alpha(k)\cos(k\eta)$, $\Sigma(\eta; \eta', k) = \int_{\boldsymbol{q}} \sigma(\boldsymbol{q}, \boldsymbol{k}) e^{-i(\omega_+ - \omega_-)(\boldsymbol{q},\boldsymbol{k})(\eta - \eta')}$

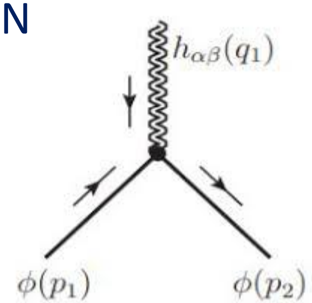
η = CONFORMAL TIME

$$\int_{\eta_0}^{\eta} d\eta' \, \Sigma(\eta; \eta', k) \, h_\alpha(\eta', \boldsymbol{k}) = \sum_\pm e^{\pm ik\eta} \int_0^{\eta-\eta_0} d\Delta\eta \int_{\boldsymbol{q}} \sigma(\boldsymbol{q}, \boldsymbol{k}) e^{-i[(\omega_+ - \omega_-)(\boldsymbol{q},\boldsymbol{k}) \pm k](\eta - \eta')}$$

$$= \sum_\pm e^{\pm ik\eta} \int_{\boldsymbol{q}} \sigma(\boldsymbol{q}, \boldsymbol{k}) \frac{\sin[(\omega_+ - \omega_- \pm k)(\eta - \eta_0)]}{\omega_+ - \omega_- \pm k}$$

- THE ONE-LOOP CAN EXHIBIT A SECULAR GROWTH (NAIVELY: $\propto \eta \propto$ a) WHEN THE CUBIC VERTEX FOR GRAVITON EMISSION/ABSORPTION IS ON-SHELL:

$$\omega_+ - \omega_- \pm k \to 0,$$



WHERE $\omega_\pm^2 = q^2 + \frac{1}{4}k^2 + (am)^2 \pm \boldsymbol{q} \cdot \boldsymbol{k}$. THIS IS A ONE-LOOP KINEMATIC CONDITION FOR ON-SHELL ABSORPTION OF ONE SCALAR AND EMISSION OF ANOTHER.

- A CAREFUL CALCULATION SHOWS: WHEN m=0, THE GROWTH IS LOGARITHMIC (IN ACCORDANCE WITH WEINBERG'S THM), see Glavan's talk.
- AN IMPORTANT QUESTION: DOES THE (THERMAL) MASS CUT-OFF THE GROWTH, AND IF YES AT WHAT SCALE?

TRANSVERSALITY & WARD IDENTITIES

- WARD IDENTITY FOR THE METRIC PERTURBATION:

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \delta g_{\mu\nu}(x)$$

$$\delta^{\alpha}_{\mu}\overline{\nabla}_{\nu}\Big(-i\big[^{\mu\nu}\Sigma^{\rho\sigma}\big]^{\text{cov}}(x;x')\Big) + \frac{i\kappa^2}{2}\bigg[\overline{g}^{\alpha(\rho}\delta^{\sigma)}_{(\mu}\Big(\overline{\nabla}_{\nu)}\frac{\delta^D(x-x')}{\sqrt{-g}}\Big) - \frac{1}{2}\delta^{(\rho}_{\mu}\delta^{\sigma)}_{\nu}\overline{g}^{\alpha\beta}\Big(\overline{\nabla}_{\beta}\frac{\delta^D(x-x')}{\sqrt{-g}}\Big)\bigg]T^{\mu\nu}(x) + \mathcal{O}(\delta g_{\alpha\beta}) = 0$$

- WARD IDENTITY FOR THE INVERSE METRIC PERTURBATION:

$$g^{\mu\nu}(x) = \bar{g}^{\mu\nu}(x) + \delta g^{\mu\nu}(x)$$

$$\overline{g}^{\alpha\mu}\overline{\nabla}_{\alpha}\Big(-i\big[_{\mu\nu}\Sigma_{\rho\sigma}\big]^{\text{cov}}(x;x')\Big) - \frac{i\kappa^2}{2}\bigg[\overline{\nabla}_{(\rho}\Big(T_{\sigma)\nu}(x)\frac{\delta^D(x-x')}{\sqrt{-g}}\Big) + \frac{1}{2}T_{\rho\sigma}(x)\overline{\nabla}_{\nu}\Big(\frac{\delta^D(x-x')}{\sqrt{-g}}\Big)\bigg] + \mathcal{O}(\delta g_{\alpha\beta}) = 0,$$

- <u>SURPRIZE I</u>: WARD IDENTITIES DEPEND ON THE REPRESENTATION OF METRIC PERTURBATION (EVEN AT THE LINEAR LEVEL)!
- <u>SURPRIZE II</u>: NOT SATISIFED FOR THE VACUUM-THERMAL PART!
- (LIKELY) SOLUTION: THE HILBERT-EINSTEIN PART CONTRIBUTES TO THE WARD IDENTITY, i.e. BY ITSELF IS NOT TRANSVERSE! (under investigation)

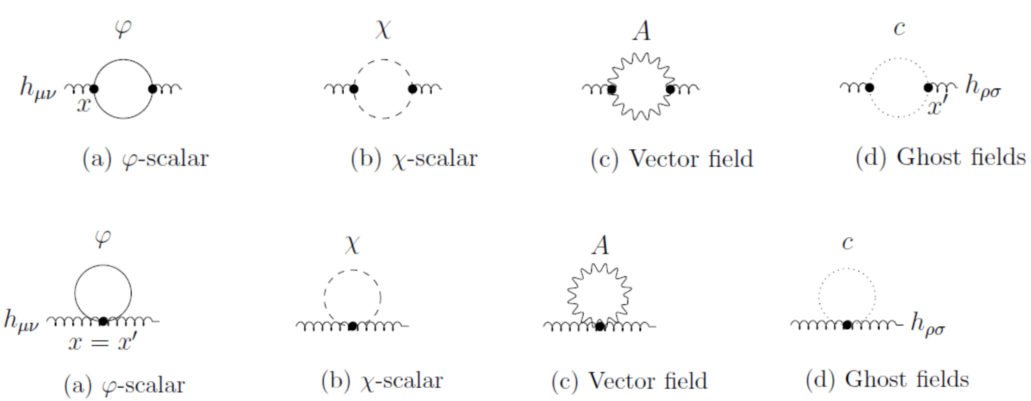
$$S^{(2)}_{\text{HE}} = \int \mathrm{d}^D x\, a^{D-2}\bigg\{\frac{1}{2}(\partial^{\rho}h^{\mu\nu})(\partial_{\mu}h_{\rho\nu}) - \frac{1}{2}(\partial^{\mu}h_{\mu\nu})(\partial^{\nu}h) + \frac{1}{4}(\partial^{\mu}h)(\partial_{\mu}h) - \frac{1}{4}(\partial^{\rho}h^{\mu\nu})(\partial_{\rho}h_{\mu\nu}) - \frac{(D-2)}{2}\mathcal{H}\delta^0_{\mu}h^{\mu\nu}(\partial_{\nu}h) + (h^2 - 2h^{\mu\nu}h_{\mu\nu})\frac{D-2}{8}\big[(D-3)\mathcal{H}^2 + 2\mathcal{H}' - a^2\Lambda_0\big] + (hh^{\mu\nu} - 2h^{\mu\rho}h^{\nu}_{\rho})\delta^0_{\mu}\delta^0_{\nu}\frac{D-2}{2}(\mathcal{H}' - \mathcal{H}^2)\bigg\}.$$

SCGP IN RADIATION IV

Abe Fennema, T.P., master thesis (2025)

- NAÏVE ESTIMATE: $\hbar G T^2 \sim \frac{H}{M_P} \sim \frac{T^2}{M_P^2} \sim 10^{-6}$ AT PREHEATING $\sim 10^{-16}$ AT ELECTROWEAK SCALE.

- WHAT HAPPENS IN THE PLASMA AT PREHEATING, WHEN PARTICLES GAIN MASS BY THE **BEH** (BROUT-ENGLERT-HIGGS) MECHANISM?
- TO UNDERSTAND THAT QUESTION WE HAVE CONSIDERED SCALAR ELECTRODYNAMICS (SQED) BOTH IN SYMMETRIC AND BROKEN PHASE).
- UP TO THIS POINT WE HAVE COMPLETED THE VACUUM PART ONLY.
- SINCE THERE ARE SOME NOTABLE RESULTS, IT IS WORTH MENTIONING THEM.
  - WE USE THE PHOTON PROPAGATOR WITH ONE PARAMETER FAMILY OF COVARIANT (FERMI → t' HOOFT) GAUGES (GAUGE PARAMETER ξ).
  - IT IS ESSENTIAL TO INCLUDE FP-GHOST CONTRIBUTIONS IN A THERMAL PLASMA.
  - THE CONTRIBUTING DIAGRAMS ARE SHOWN IN THE FIGURE BELOW
- WE HAVE COMPUTED THE SELF ENERGY BOTH IN THE SYMMETRIC AND BROKEN PHASES.



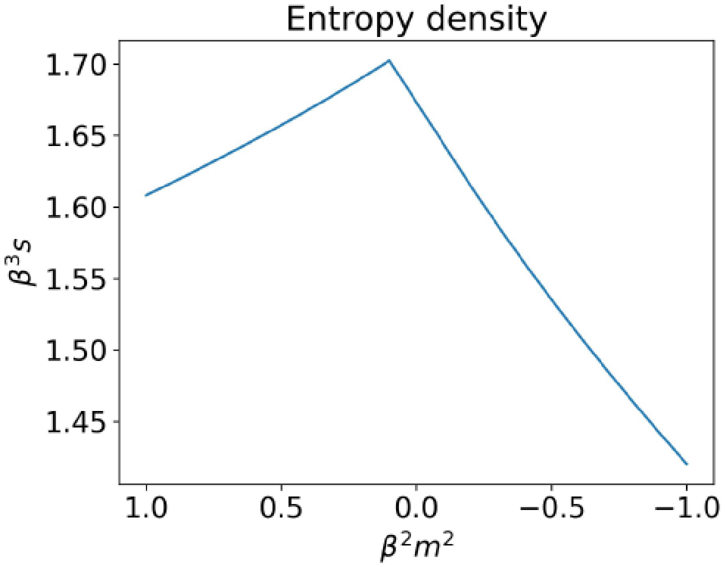
RESULTS:

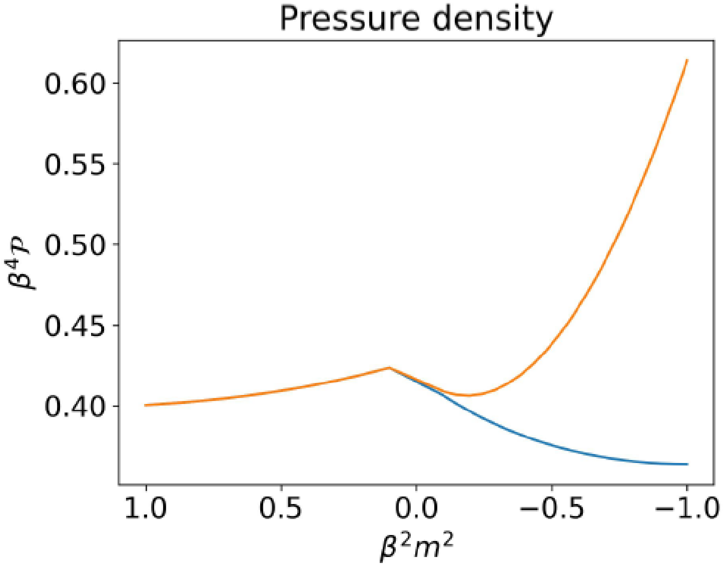
- THE SELF ENERGY IS INDEPENDENT ON THE PHOTON GAUGE PARAMETER ξ
- TRANSVERSALITY OF THE SELF-ENERGY DOES NOT CHANGE AT THE TRANSITION, EVEN THOUGH THE VACUUM ENERGY DOES CHANGE!

SCGP IN RADIATION V

Abe Fennema, T.P., master thesis (2025)

- PLOTTING THE ENTROPY DENSITY AND PRESSURE ACCORSS THE TRANSITION WITH OUR RENORMALIZED ENERGY MOMENTUM TENSOR GIVES:





SCGP IN RADIATION VI

Abe Fennema, T.P., master thesis (2025)

- THE EXPLICIT FORM OF THE RENORMALIZED SELF-ENERGY AND THE COUNTERTERMS

$$i[_{\mu\nu}\Sigma^{ren}_{\rho\sigma}]^{be}(x,x'') = \frac{\kappa^2}{3840\pi^2 a^4}\Bigg\{\Bigg[-(16a^4m^4+12a^2m^2\partial^2)\Pi_m i\delta^4(x-x'')$$
$$+(16a^4m^4+32\partial^2a^2m^2+6\partial^4)M_m(x,x'')-8\partial^2 M_0(x,x'')\Bigg]P_{\mu\nu}P_{\rho\sigma}$$
$$+\Bigg[-\left(16a^4m^4+2a^2m^2\partial^2\right)\Pi_m i\delta^4(x-x'')$$
$$+(\partial^2-4a^2m^2)^2M_m(x,x'')+12\partial^2M_0(x,x'')\Bigg]2P_{\mu(\rho}P_{\sigma)\nu}\Bigg\}$$
$$-\frac{\kappa^2}{a^4}\Bigg\{c_{1f}\frac{2}{3}\left(P_{\mu(\rho}P_{\sigma)\nu}-\frac{1}{3}P_{\mu\nu}P_{\rho\sigma}\right)\partial^4+2c_{2f}P_{\mu\nu}P_{\rho\sigma}\partial^4+\frac{1}{480\pi^2}P_{\mu\nu}P_{\rho\sigma}a^2m^2\partial^2$$
$$+\frac{1}{2}a^2m^2c^{\Phi}_{3f}(P_{\mu(\rho}P_{\sigma)\nu}-P_{\mu\nu}P_{\rho\sigma})\partial^2+\frac{1}{240\pi^2}(2P_{\mu(\rho}P_{\sigma)\nu}+P_{\mu\nu}P_{\rho\sigma})a^4m^4$$
$$+\frac{1}{4}a^4m^4\left(c_{4f}-\frac{1}{32\pi^2}\Pi_m\right)(\eta_{\mu\nu}\eta_{\rho\sigma}+2\eta_{\mu(\rho}\eta_{\sigma)\nu})\Bigg\}i\delta^4(x-x').$$

$$i[^{ct}_{\mu\nu}\Sigma_{\rho\sigma}](x,x') = -\frac{\kappa^2}{a^D}\Bigg\{c_1\frac{2(D-3)}{D-2}\left(P_{\mu(\rho}P_{\sigma)\nu}-\frac{1}{D-1}P_{\mu\nu}P_{\rho\sigma}\right)\partial^4$$
$$+2c_2P_{\mu\nu}P_{\rho\sigma}\partial^4+\frac{1}{2}a^2\left(c_3^{\Phi}m^2+a^{2-D}\bar{\phi}^2c_3^{\phi}\right)(P_{\mu(\rho}P_{\sigma)\nu}-P_{\mu\nu}P_{\rho\sigma})\partial^2$$
$$+\frac{1}{4}(\eta_{\mu\nu}\eta_{\rho\sigma}+2\eta_{\mu(\rho}\eta_{\sigma)\nu})a^4m^4c_4\Bigg\}i\delta^D(x-x'). \qquad (3.98)$$

$$c_1 = \frac{1}{(16\pi)^2}\frac{D-2}{(D-3)(D^2-1)}\left[2+(2D^2-3D-8)\right]\frac{\mu^{D-4}}{D-4}+c_{1f}$$
$$= \frac{1}{(16\pi)^2}\frac{(D-2)(2D^2-3D-6)}{(D-3)(D^2-1)}\frac{\mu^{D-4}}{D-4}+c_{1f},$$
$$c_2 = \frac{1}{(16\pi)^2}\frac{(D-2)^2}{(D-1)^2}\frac{\mu^{D-4}}{D-4}+c_{2f},$$
$$c_3^{\Phi} = \frac{8}{(16\pi)^2}\frac{-10}{D^2-1}\frac{\mu^{D-4}}{D-4}+c^{\Phi}_{3f},$$
$$c_3^{\phi} = -\frac{8\lambda}{(16\pi)^2(D^2-1)m^2}\left(10m^2-5(-2m^2)-\frac{8D^2-28D-76}{4}M^2\right)\frac{\mu^{D-4}}{D-4}+c_{3f},$$
$$c_4 = \frac{1}{4D\pi^2}\frac{\mu^{D-4}}{D-4}+c_{4f},$$

and we fix δm^2 as,

$$\delta m^2 = -\frac{\lambda}{2D\pi^2m^2}\left(2m^4-(-2m^2)^2-(D-1)M^4\right)\frac{\mu^{D-4}}{D-4}+\delta m_f^2.$$

SCGP IN RADIATION VII

Abe Fennema, T.P., master thesis (2025)

- THE EXPLICIT FORM OF THE RENORMALIZED SELF-ENERGY AFTER THE (EW) TRANSITION:

$$
\begin{aligned}
i[{}_{\mu\nu}\Sigma^{ren}_{\rho\sigma}]^{af}(x,x'') = \frac{\kappa^2}{7680\pi^2 a^4}\Bigg\{\Bigg[&-\left(16a^4(-2m^2)^2 + 12a^2(-2m^2)\partial^2\right)\Pi_{\sqrt{-2m^2}}\, i\delta^4(x-x'') \\
&+ \left(16a^4(-2m^2)^2 + 32\partial^2 a^2(-2m^2) + 6\partial^4\right)M_{\sqrt{-2m^2}}(x,x'') + \left(4a^2M^2\partial^2 - 48a^4M^4\right)\Pi_M\, i\delta^4(x-x'') \\
&+ \left(-2\partial^4 - 64\partial^2 a^2 M^2 + 48a^4M^4\right)M_M(x,x'')\Bigg]P_{\mu\nu}P_{\rho\sigma} \\
&+ \Bigg[-\left(16a^2(-2m^2) + 2\partial^2\right)\Pi_{\sqrt{-2m^2}}\, i\delta^4(x-x'') + \left(\partial^2 - 4a^2(-2m^2)\right)^2 M_{\sqrt{-2m^2}}(x,x'') \\
&+ \left(-26a^2M^2\partial^2 - 48a^4M^4\right)\Pi_M\, i\delta^4(x-x'') \\
&+ \left(13\partial^4 + 56a^2M^2\partial^2 + 48a^4M^4\right)M_M(x,x'')\Bigg]2P_{\mu(\rho}P_{\sigma)\nu}\Bigg\} \\
&- \frac{\kappa^2}{a^4}\Bigg\{c_{1f}\frac{2}{3}\left(P_{\mu(\rho}P_{\sigma)\nu} - \frac{1}{3}P_{\mu\nu}P_{\rho\sigma}\right)\partial^4 + 2c_{2f}P_{\mu\nu}P_{\rho\sigma}\partial^4 \\
&+ \frac{1}{2}a^2\left(c^{\Phi}_{3f}m^2 + a^{-2}\bar{\phi}^2 c^{\phi}_{3f}\right)\left(P_{\mu(\rho}P_{\sigma)\nu} - P_{\mu\nu}P_{\rho\sigma}\right)\partial^2 + \frac{1}{960\pi^2}P_{\mu\nu}P_{\rho\sigma}\left(a^2(-2m^2) + 3a^2M^2\right)\partial^2 \\
&+ \frac{1}{480\pi^2}\left(2P_{\mu(\rho}P_{\sigma)\nu} + P_{\mu\nu}P_{\rho\sigma}\right)\left(a^4(-2m^2)^2 + 3a^4M^4\right) \\
&+ \frac{1}{4}a^4\left(\eta_{\mu\nu}\eta_{\rho\sigma} + 2\eta_{\mu(\rho}\eta_{\sigma)\nu}\right)\Bigg[m^4\left(c_{4f} - \frac{1}{32\pi^2}\Pi_m\right) \\
&+ a^{-2}\bar{\phi}^2\left(-\frac{1}{4}\delta m_f^2 + m^2\left(-\frac{1}{4} + \lambda\left(\frac{1}{32\pi^2}\Pi_m - \frac{1}{16\pi^2}\Pi_{\sqrt{-2m^2}}\right) - \frac{3q^4}{64\pi^2\lambda}\Pi_M\right)\right)\Bigg]\Bigg\}i\delta^4(x-x').
\end{aligned}
$$

CONCLUSIONS AND OUTLOOK

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- PRECISE MAPPINGS OF THE CMB AND LSS DEMAND A BETTER UNDERSTANDING OF THE CREATION AND EVOLUTION OF GRAVITATIONAL PERTURBATIONS IN THE CONTEXT OF INFLATION AND BIG BANG MODEL (RADIATION & MATTER ERA).

- IN THIS TALK I HAVE ARGUED THAT SUCH A THEORY IS A SEMICLASSICAL THEORY OF GRAVITATIONAL PERTURBATIONS (**SCGP**).
- OUR UNDERSTANDING OF THIS THEORY IS AT BEST RUDIMENTARY.
- SEVERAL INSTANCES THAT HAVE BEEN STUDIED POINT AT POSSIBLE SECULAR EFFECTS IN TIME, WHICH MIGHT SIGNIFICANTLY ENHANCE THE TREE-LEVEL AMPLITUDE OF GRAVITATIONAL PERTURBATIONS, BOTH IN INFLATION AND RADIATION.