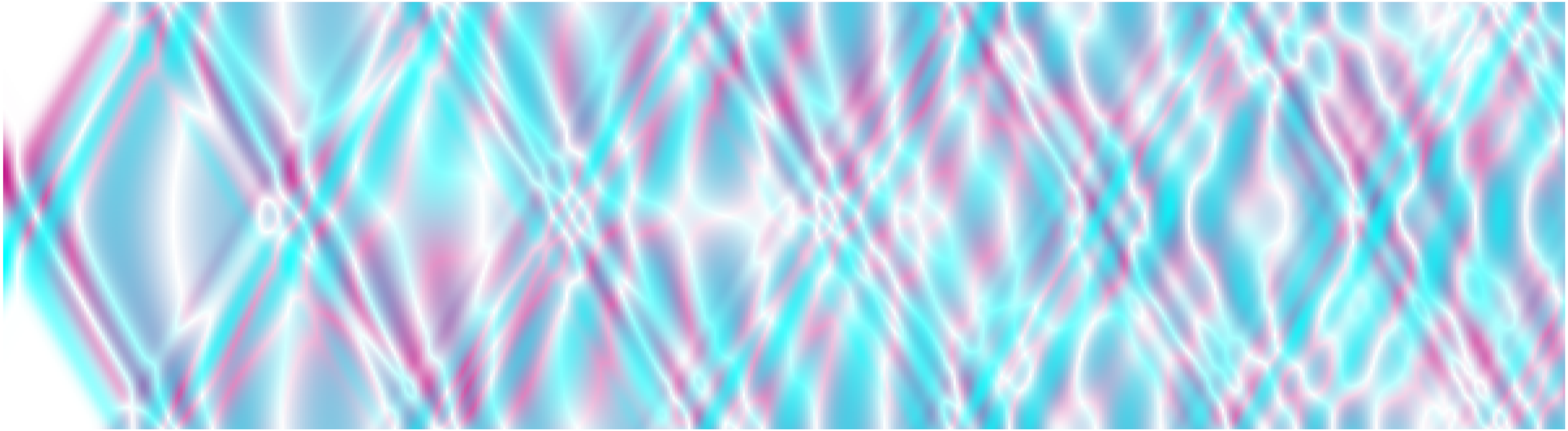

Ghostly interactions in classical field theory: From scalar models to black-hole binaries

Aaron Held

Philippe Meyer Junior Research Chair
École Normale Supérieure

April 11, 2025: CosmoGrav 2025
CEICO, FZU, Prague

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Motivation: Gravitational Effective Field Theories

Part I: Ghosts in classical field theory with Cédric Deffayet, Shinji Mukhoyama, and Alexander Vikman

Part II: Nonlinear evolution & black hole binaries
with Pau Figueras and Áron Kovács with Hyun Lim

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Motivation: Gravitational Effective Field Theories

The Effective Field Theory Framework …

Assuming a given

(i)	IR **field content**		**metric ***
(ii)	IR **symmetries**		**Lorentz invariance**
(iii)	**expansion** scale/scheme		**derivative / curvature**

we expand the effective action in all possible operators.

Within the **regime of validity** of the EFT
the effect of any unknown UV physics
is captured in the values of the EFT coefficients.

*hidden additional gravitational degrees of freedom

… systematically captures modifications of GR.

Joint derivative/curvature expansion ...

$$\mathcal{L}^{(1)}_{\rm EFT} = M_{\rm Pl}^2\, R$$

$$\mathcal{L}^{(2)}_{\rm EFT} = \Big[\alpha_0 R_{ab}R^{ab} - \beta_0\, R^2\Big]$$

After reduction via
(i) index symmetries
(ii) geometric identities
(iii) 4D-specific identities (e.g. Gauss-Bonnet)
see Fulling CQG 9 (1992); Martin-Garcia, Yllanes, Portugal, CPC 179 (2008)

$$\mathcal{L}^{(3)}_{\rm EFT} = \frac{1}{M_{\rm Pl}^2}\Big[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab} + \delta_{3,1}\, C_{abcd}C^{abcd}R + \delta_{3,2}\, C_{abcd}R^{ac}R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab}R^{ab}R + \delta_{3,5}\, R^3\Big]$$

$$\mathcal{L}^{(4)}_{\rm EFT} = \frac{1}{M_{\rm Pl}^4}\Big[\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R + \gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^*C^{abcd})^2 + \dots\Big]$$

... before field redefinitions.

Joint derivative/curvature expansion ...

order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M_{\mathrm{Pl}}^2\, R$$

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\alpha_0 R_{ab} R^{ab} - \beta_0\, R^2\right]$$

$$\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M_{\mathrm{Pl}}^2}\Big[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \boxed{\gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}}$$

$$+ \delta_{3,1}\, C_{abcd} C^{abcd} R + \delta_{3,2}\, C_{abcd} R^{ac} R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab} R^{ab} R + \delta_{3,5}\, R^3\Big]$$

Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M_{\mathrm{Pl}}^4}\Big[\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R + \boxed{\gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^{*}C^{abcd})^2} + \dots\Big]$$

Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

... after field redefinitions.

Joint derivative/curvature expansion ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M^2_{\mathrm{Pl}}\, R$$

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$$+ \delta_{3,1}\, C_{abcd} C^{abcd} R + \delta_{3,2}\, C_{abcd} R^{ac} R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab} R^{ab} R + \delta_{3,5}\, R^3\Big]$$

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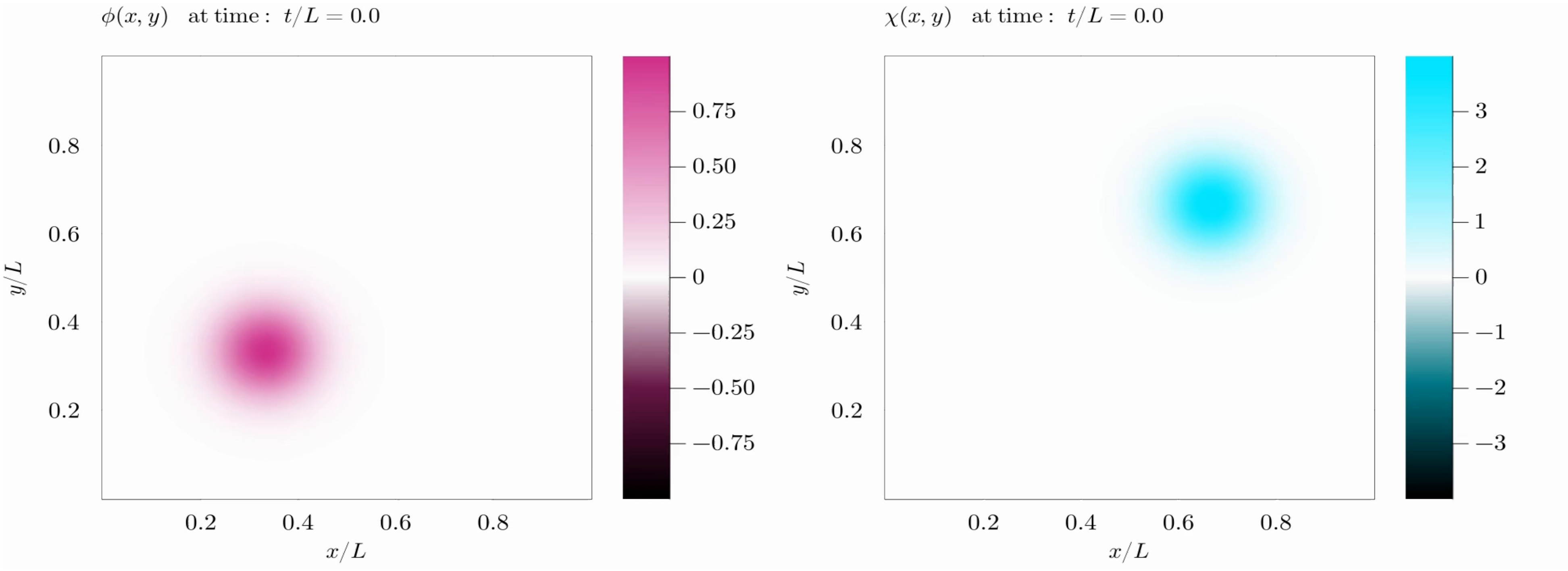
can remove any term containing Ricci variables

... after field redefinitions.



Part I: Ghosts in classical field theory

with C. Deffayet, S. Mukohyama and A. Vikman, to appear

For point-particle models, see Cédric's talk as well as
Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

Point-particle models can be stable if the “long-range” potential is dominated by stable self-interactions.

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

Classical field theories do not decay instantaneously and can **exhibit longlived motion.**

Deffayet, Held, Mukohyama, Vikman, to appear

~~All non-degenerate **higher-derivative classical point-particle** theories **exhibit runaway** solutions.~~

~~Ok, but **field theories** will still **decay instantaneously** because of an **infinite phase-space volume** at high energy.~~

Ok, but **quantised field theories** will **decay instantaneously**, right?

Part I: Ghosts in classical field theory

with C. Deffayet, S. Mukohyama and A. Vikman, to appear

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Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

Opposite-sign kinetic terms ...

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

The **Helmhotz-Lagrangian** (equivalent 2nd-order Lagrangian) of non-degenerate higher-derivative theories exhibits **opposite-sign kinetic terms.**

Urries, Julve J. Phys. A 31 (1998)

- e.g., Lagrangian

$$\mathcal{L} = -\frac{1}{2}\phi\left[\Box + \mathrm{m}_\phi^2\right]\phi - \frac{\sigma}{2}\chi\left[\Box + \mathrm{m}_\chi^2\right]\chi - \mathrm{V}(\phi,\,\chi)$$

$\sigma = +1$: non-ghostly
$\sigma = -1$: ghostly

- with field equations

$$\left[\Box + \mathrm{m}_\phi^2\right]\phi = -\partial_\phi \mathrm{V}$$

$$\left[\Box + \mathrm{m}_\chi^2\right]\chi = -\sigma\,\partial_\chi \mathrm{V}$$

(non-)ghostly nature ($\sigma = \pm 1$)
does not affect the principal part

... do not obstruct from (local) well-posedness.

Opposite-sign kinetic terms ...

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

The **Helmhotz-Lagrangian** (equivalent 2nd-order Lagrangian) of non-degenerate higher-derivative theories exhibits **opposite-sign kinetic terms.**

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σ = + 1: non-ghostly
σ = – 1: ghostly

- with field equations

$$\left[\Box + \mathrm{m}_\phi^2\right]\phi = -\partial_\phi \mathrm{V}$$

$$\left[\Box + \mathrm{m}_\chi^2\right]\chi = -\sigma\,\partial_\chi \mathrm{V}$$

(non-)ghostly nature (σ = ± 1)
does not affect the principal part

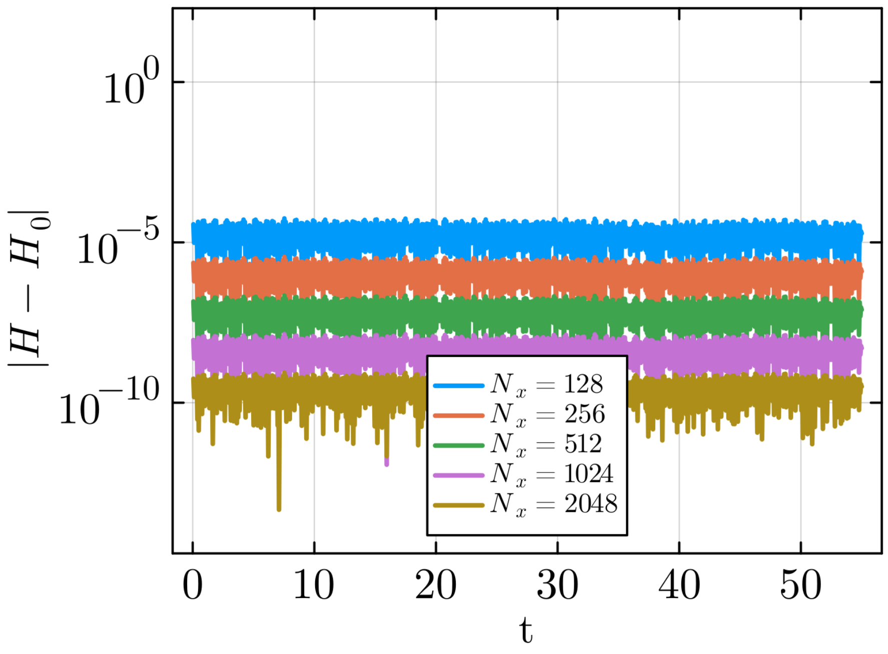
... **cannot** lead to instantaneous classical decay.

Numerical solution ...

- using DifferentialEquations.jl
  https://github.com/aaron-hd/ghostlyPDE_1D
- julia-based PDE solver
  Rackauckas, Nie, JORS 5 (2017)

DifferentialEquations.jl

- 4th order finite differencing (FD) in space
- 4th order Runge Kutta (RK4) timestep
- confirm convergence of the Hamiltonian



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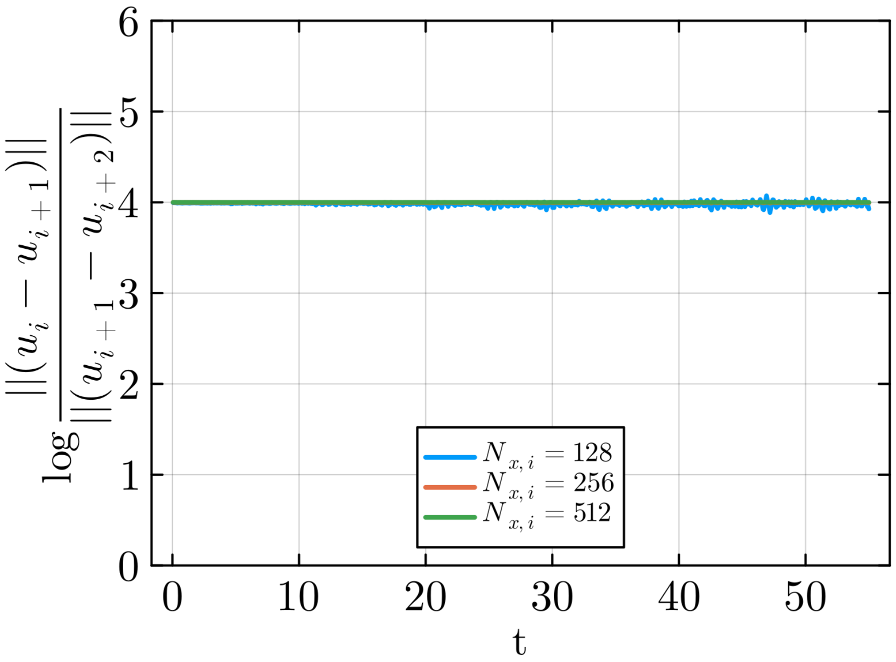
DifferentialEquations.jl

- julia-based PDE solver
  Rackauckas, Nie, JORS 5 (2017)

- 4th order finite differencing (FD) in space
- 4th order Runge Kutta (RK4) timestep

- confirm convergence of the Hamiltonian
- automatic self-convergence tests

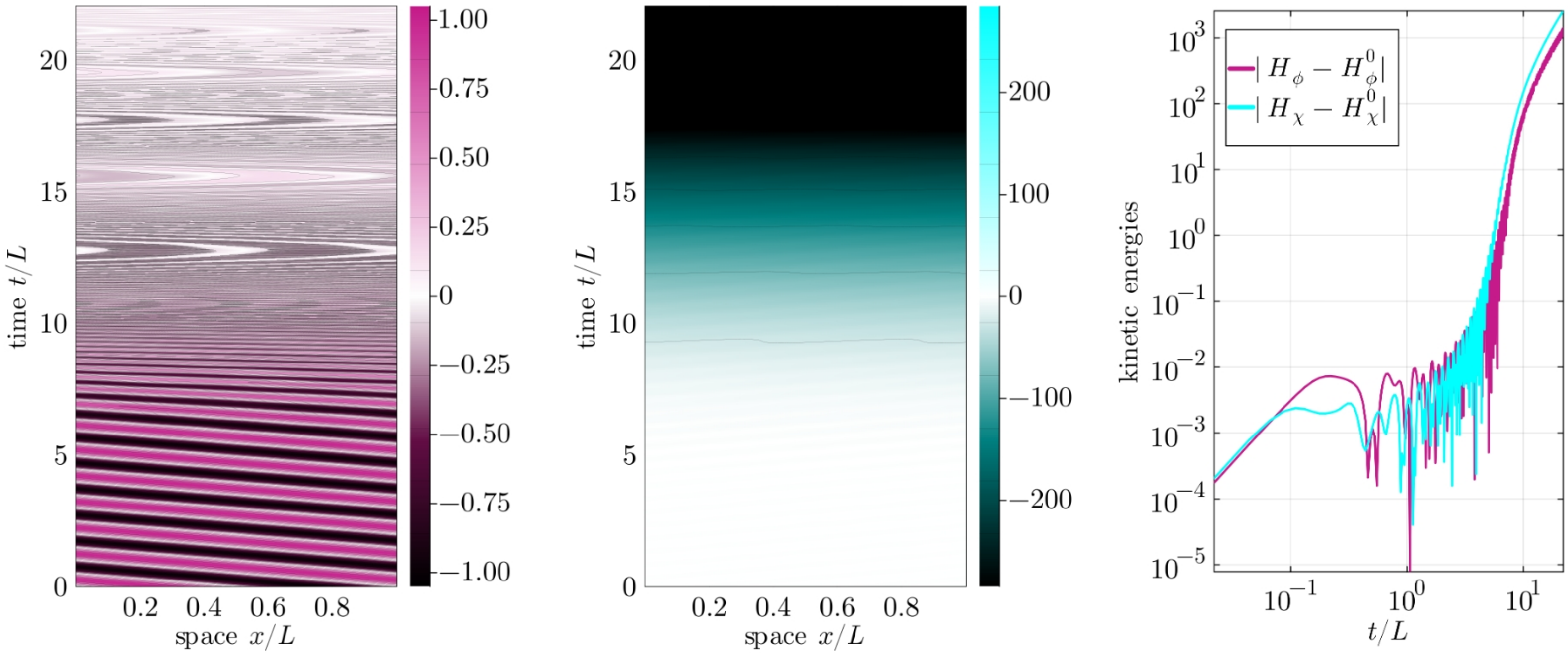


… converges to the continuum field theory.

Unquenched instability …

Deffayet, Held, Mukohyama, Vikman, to appear

$$V = \lambda_{nm}\, \phi^n \chi^m$$



(1+1)D Simulation converges to the solution of the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters

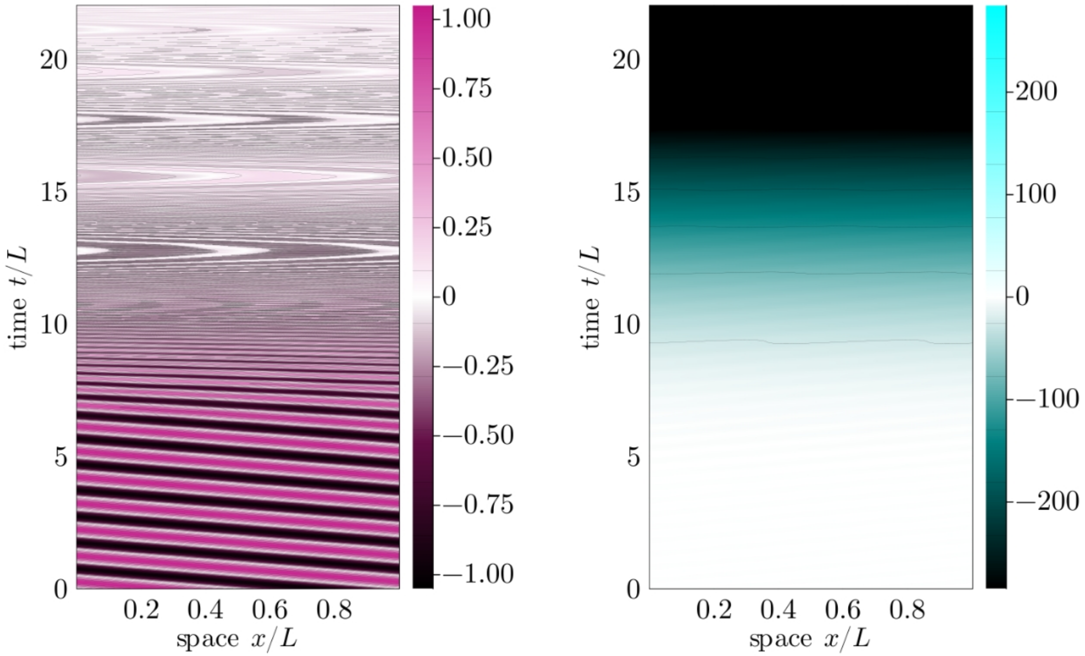
- n = m = 2
- $\lambda L^2 = 1$;
- $m_\phi L = m_\chi L = 0$;

plane-wave initial data

Unquenched instability ...

Deffayet, Held, Mukohyama, Vikman, to appear

$$V = \lambda_{nm}\, \phi^n \chi^m$$



$$\Box\phi = -\left(m_\phi^2 + 2\,\lambda_{22}\,\chi^2\right)\phi \equiv -m_{\phi,\text{eff}}^2\,\phi$$

$$\Box\chi = -\left(m_\chi^2 + 2\,\sigma\,\lambda_{22}\,\phi^2\right)\chi \equiv -m_{\chi,\text{eff}}^2\,\chi$$

- no ghost: both effective masses positive
- with ghost: one effective mass positive
  one effective mass negative

... can be benign.

Higher frequencies ...

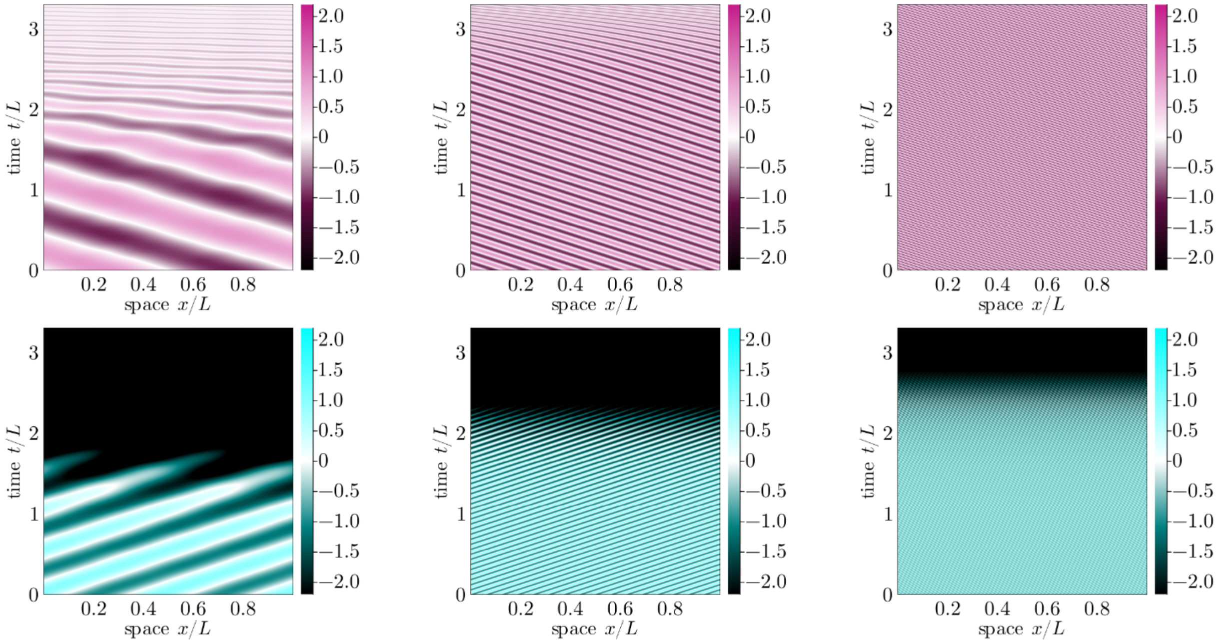
Deffayet, Held, Mukohyama, Vikman, to appear

$$V = \lambda_{nm}\, \phi^n \chi^m$$

$$\partial_t^2 \phi = -\left(k_\phi^2 + m_\phi^2 + \lambda\, \chi^2\right) \phi$$

$$\partial_t^2 \chi = -\left(k_\chi^2 + m_\chi^2 + \sigma\, \lambda\, \phi^2\right) \chi$$

- plane-wave approximation
- **high frequency dominates potential**, both for the non-ghost and for the ghost case



... are more stable, not less stable.

Opposite-sign kinetic terms ...

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

$$V_{LV}^{(4)}(x,y) = \left(\frac{m_\phi^2}{2} - \frac{m_\chi^2}{2}\right)(\phi^2-\chi^2)^2 + \mathcal{C}_4(\phi^4-\chi^4) + \mathcal{C}_4(\phi^2-\chi^2)^3$$

Increasingly longlived for:

- lower initial amplitude
- higher initial frequency

initial data parameters

- weaker ghostly coupling
- larger masses

model parameters

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Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

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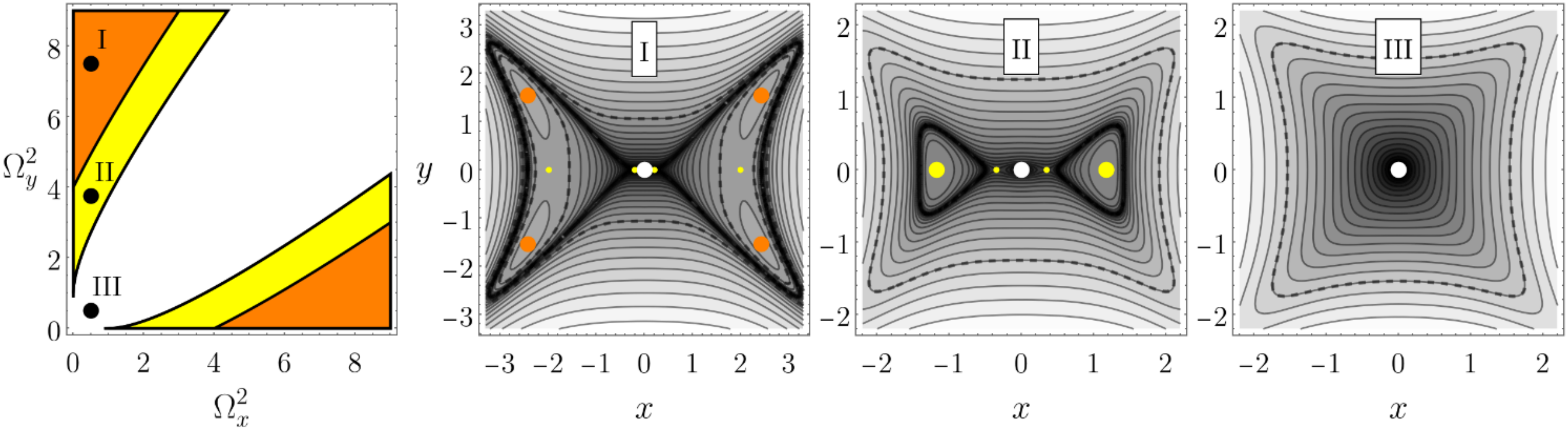

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(1+1)D Simulation converges to the solution of the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters setup in trivial vacuum

$$\boxed{m_\phi^2 \equiv m_\chi^2 \equiv m^2}$$

Various initial data families

Opposite-sign kinetic terms ...

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

$$V^{(4)}_{\mathrm{LV}}(x,y) = \left(\frac{m_\phi^2}{2} - \frac{m_\chi^2}{2}\right)(\phi^2-\chi^2)^2 + \mathcal{C}_4(\phi^4-\chi^4) + \mathcal{C}_4(\phi^2-\chi^2)^3$$

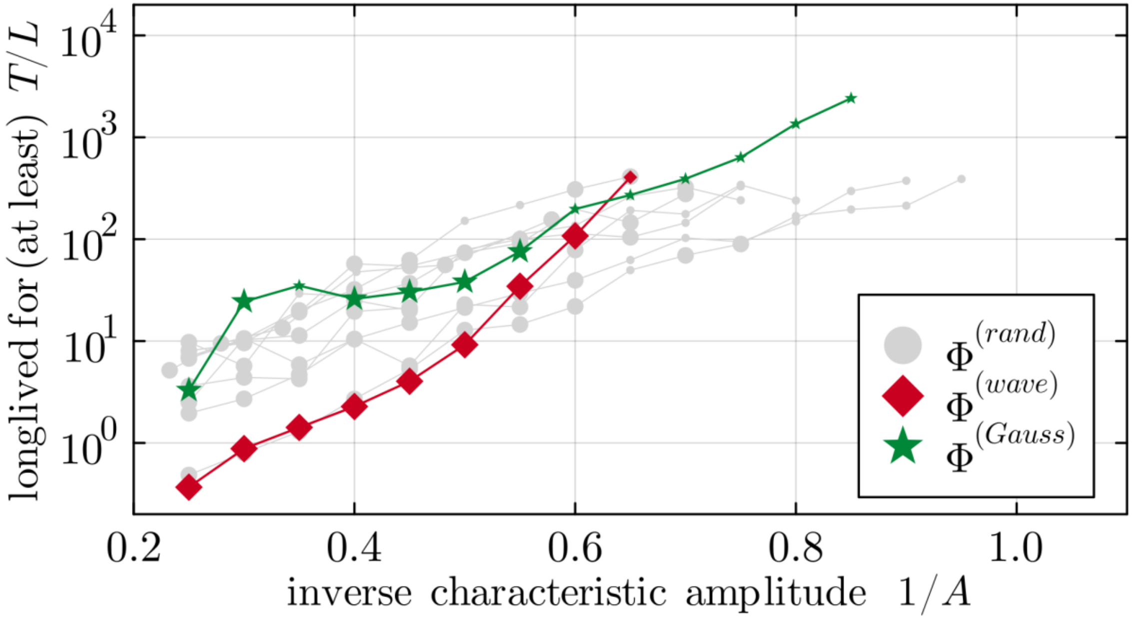
Increasingly longlived for:

- $\boxed{\text{lower initial amplitude}}$
- higher initial frequency

initial data parameters

- weaker ghostly coupling
- larger masses

model parameters



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Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

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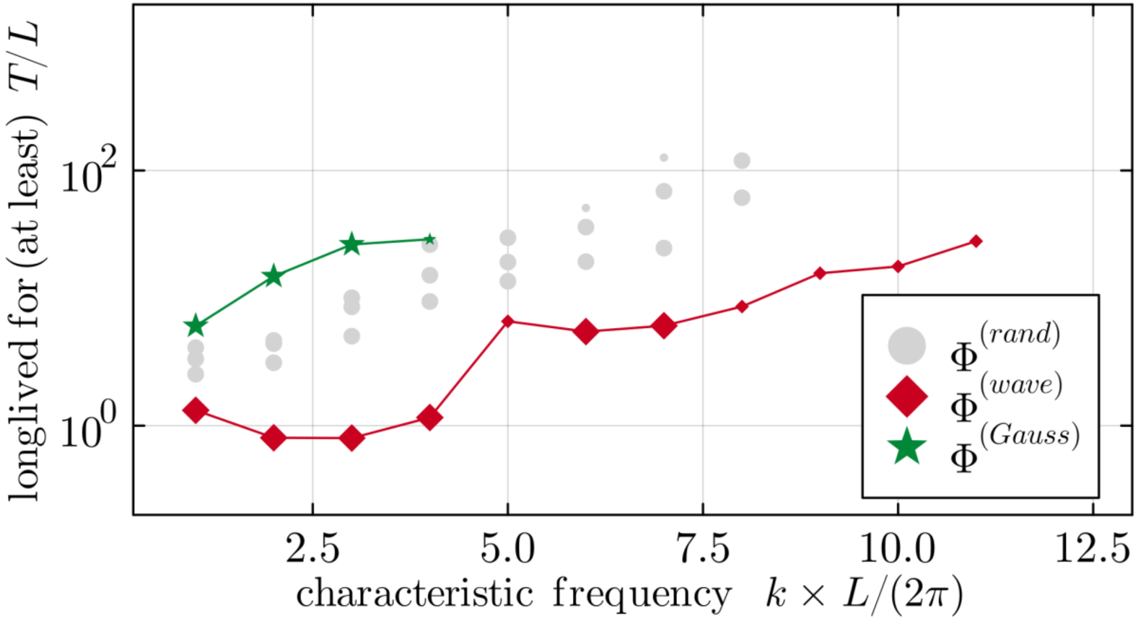

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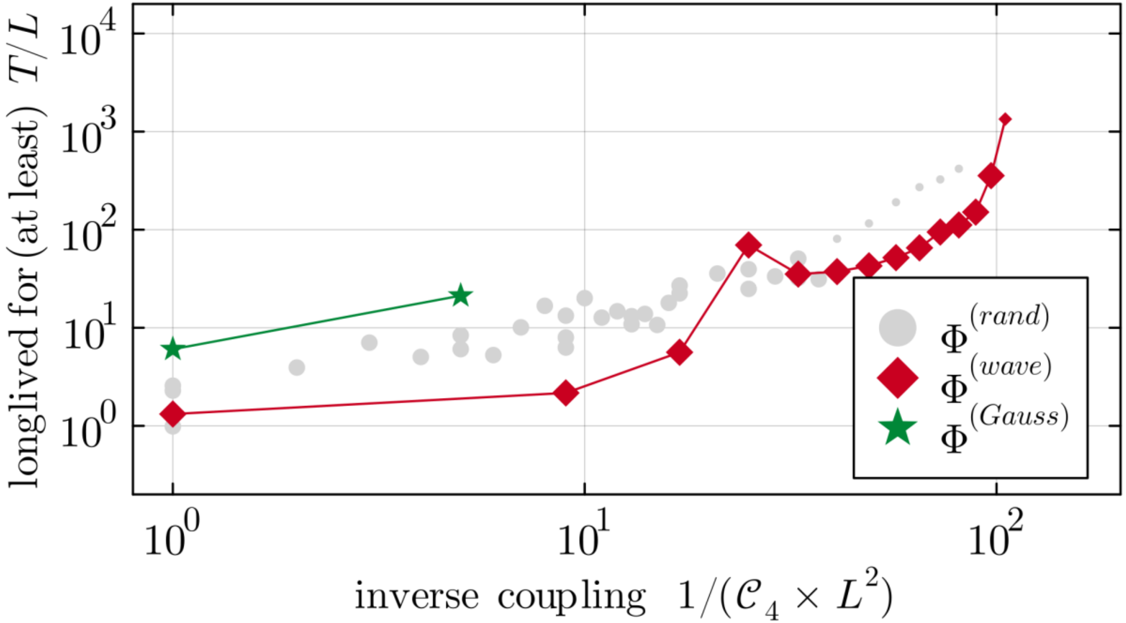

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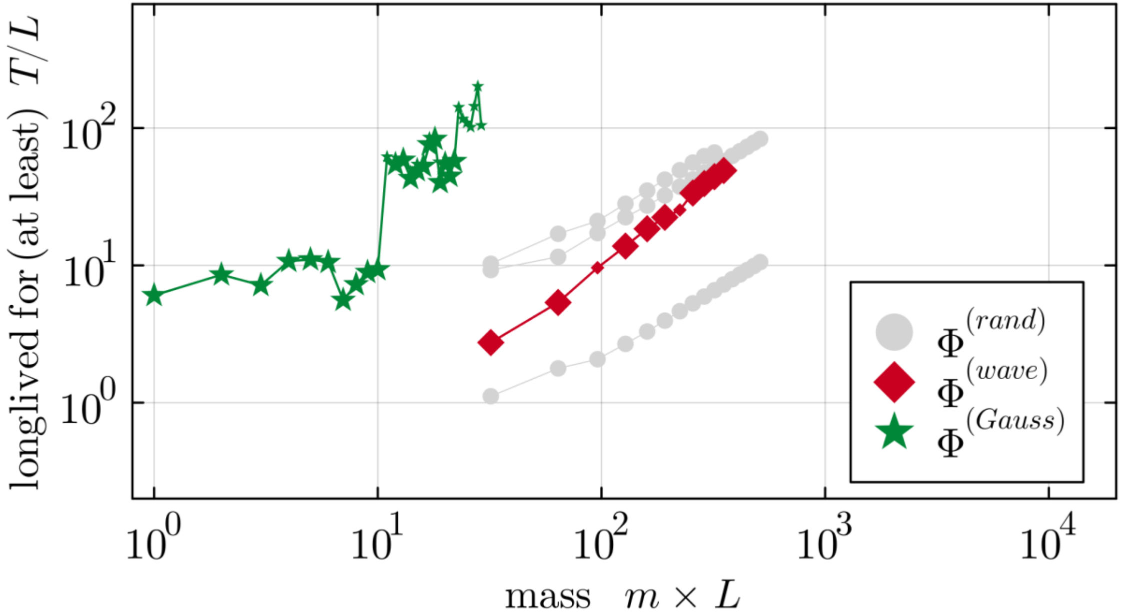
- $\boxed{\text{larger masses}}$

model parameters



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initial data parameters

- weaker ghostly coupling
- larger masses

model parameters

Deffayet, Held, Mukohyama, Vikman, to appear (see also JCAP 11 (2023) 031)

... admit for (arbitrarily) long-lived motion.

Opposite-sign kinetic terms …

$$V = \lambda\, \phi^2 \chi^2$$

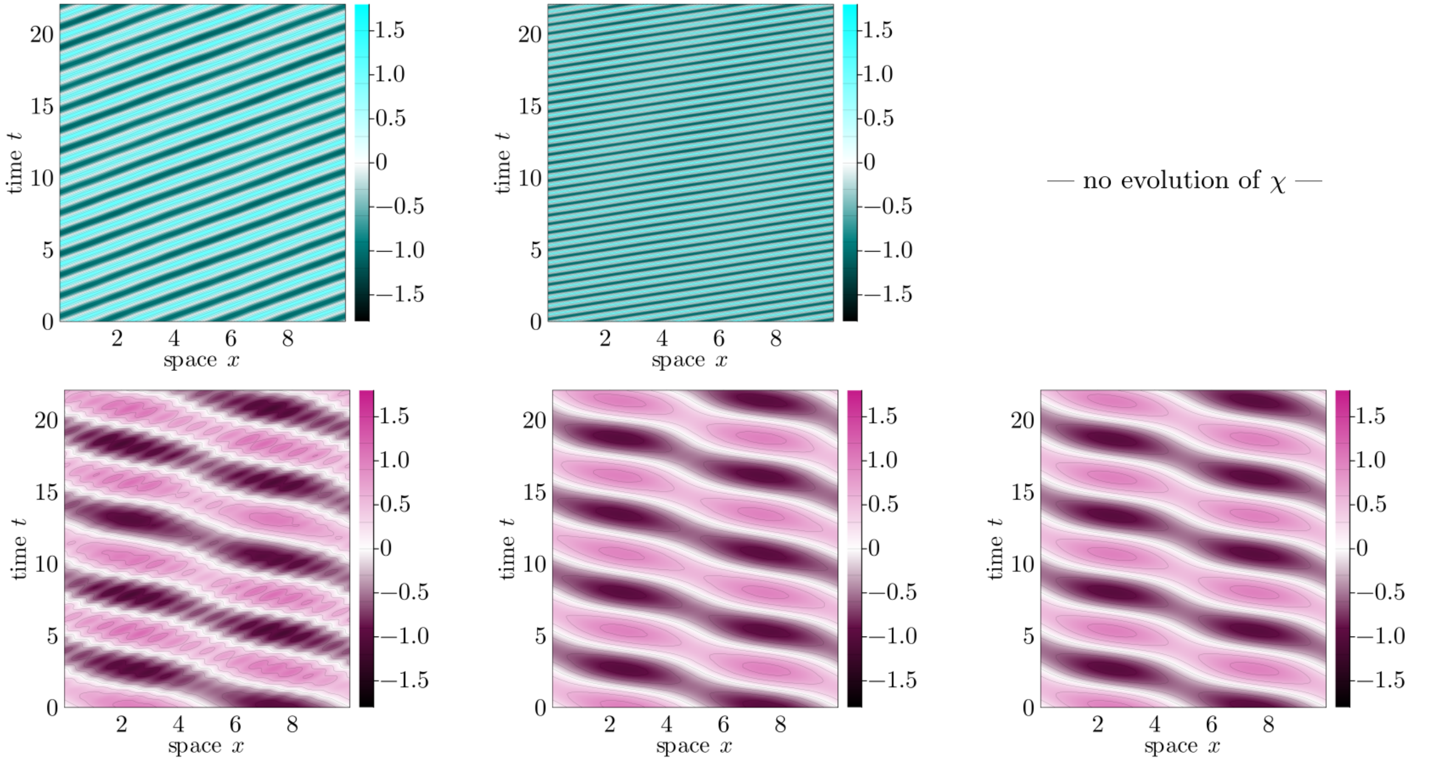
(1+1)D Simulation converges to the solution of the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters

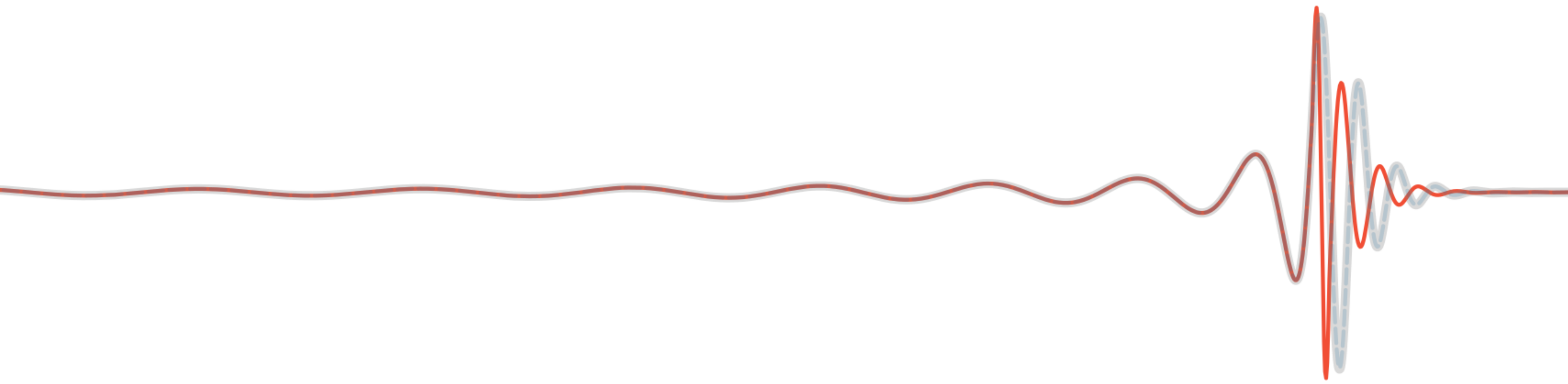
- $\lambda = 1$;
- $m_\phi = 0$;
- $m_\chi = 3$; $m_\chi = 10$; $m_\chi = \infty$ (from left to right)

Plane-wave initial data



Deffayet, Held, Mukohyama, Vikman, to appear (see also JCAP 11 (2023) 031)

… can effectively decouple if sufficiently heavy.

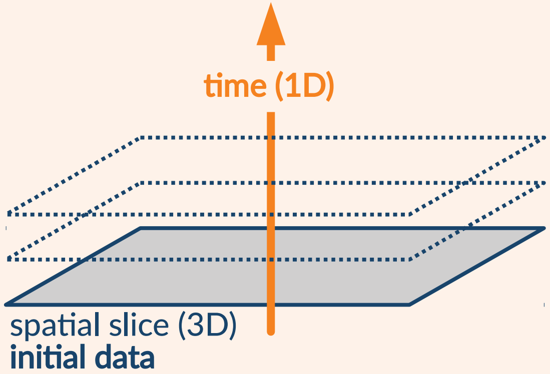


Part II: Nonlinear evolution & black hole binaries

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
Held, Lim, 2503.13428

A well-posed initial value problem (IVP) …



“ An initial value problem is well-posed if a solution “

- **exists**
- is **unique**
- and depends **continuous**ly on the initial data

… for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat ‘52

(3+1) numerical evolution
Pretorius ‘05; BSSN ‘87-’99; Sarbach et.Al ‘02-’04

… and for Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim, PRD 104 (2021) 8

Held, Lim, PRD 108 (2023) 10;
Cayuso ‘23; East, Siemonsen ‘23

… and for the EFT (at any order)

Figueras, **Held**, Kovacs, 2407.08775

General Relativity (in harmonic gauge) …

- Gauge potential: $F^a \equiv -g^{cd}\Gamma^a_{cd}$
- Ricci curvature: $R_{ab} \sim \Box g_{ab} + g_{c(a}\nabla_{b)}F^c + \mathcal{O}(g, \partial g) = 0$
- In **harmonic gauge**, i.e., $F^a = 0$
  the vacuum Einstein equations, i.e., $R_{ab} = 0$
  are of **wave-like form**.

For constraint propagation see
Choquet-Bruhat ‘52

… is already in wave-like form.

Quadratic Gravity …

Held, Lim, PRD 104 (2021) 8

- recall $\mathcal{L} = M_{Pl}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$

Stelle, PRD 16 (1977) 953-969
Noakes, JMP 24, 1846 (1983)
Held, Lim, PRD 104 (2021) 8

2nd-order variables

$$\Box g_{ab} \sim R_{ab} \equiv S_{ab} + \frac{1}{4} g_{ab} R$$

massless spin-2 (graviton)

$$\Box R = m_0^2 R$$

massive spin-0 (scalar)

vanishes for equal masses:

$$\Box S_{ab} = \boxed{-\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)(\nabla_a \nabla_b R)} - 2\, S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

massive spin-2 (ghost)

- For equal masses, the 2nd-order field equations of Quadratic Gravity are of wave-like form.

Quadratic Gravity ...

Held, Lim, PRD 104 (2021) 8

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2nd-order variables

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab} R$$

massless spin-2 (graviton)

$$\Box R \overset{2|0}{=} m_0^2 R$$

massive spin-0 (scalar)

$$\Box S_{ab} \overset{3|1}{=} -\frac{1}{3} \left(\frac{m_2^2}{m_0^2} - 1 \right) (\nabla_a \nabla_b R) - 2 S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

massive spin-2 (ghost)

- For equal masses, the 2nd-order field equations of Quadratic Gravity are of wave-like form.
- For unequal masses, one can still find suitable Leray weights.

... admits wave-like 2nd order field equations.

Cubic Gravity (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- recall $\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M^2_{\mathrm{Pl}}}\left[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}\right]$

order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab}\, R$$

$$\Box C_{abde} \overset{3|1}{=} \mathcal{O}^{C}_{abde}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

$$\Box R \overset{2|0}{\equiv} R^{(1)}$$

$$\Box S_{ab} \overset{2|0}{\equiv} S^{(1)}_{ab}$$

vanishes for equal mass: $\alpha_1 = 2\beta_1$

$$\Box R^{(1)} \overset{3|1}{\equiv} \mathcal{O}^{R}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

$$\Box S^{(1)}_{ab} \overset{3|1}{\equiv} \boxed{\left(1 - \frac{2\beta_1}{\alpha_1}\right)\left(\frac{1}{4} g_{ab}\,\Box - \nabla_a \nabla_b\right) R^{(1)}} + \mathcal{O}^{S}_{ab}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

... admits wave-like 2nd order field equations.

Higher order EFT (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}^{(n)}_{\mathrm{reg}} = \sum_{k=0}^{n} \left[\alpha_k\, R^{ab}\,\Box^k\, R_{ab} - \beta_k\, R\,\Box^k\, R\right]$ with $\alpha_n = 2\beta_n$

order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab}\, R$$

$$\Box C_{abde} \overset{3|1}{=} \mathcal{O}^{C}_{abde}(\partial C,\ \partial\partial S,\ \partial\partial R)$$

$$\Box R^{(k)} \overset{k+2|k}{\equiv} R^{(k+1)} \qquad \forall 0 \leq k < n$$

$$\Box S^{(k)}_{ab} \overset{k+2|k}{\equiv} S^{(k+1)}_{ab} \qquad \forall 0 \leq k < n$$

$$\Box R^{(n)} \overset{n+2|n}{\equiv} \mathcal{O}^{R}(\partial^{n-1}C,\ \partial^{n-k}S^{(k)},\ \partial^{n-k}R^{(k)})$$

$$\Box S^{(n)}_{ab} \overset{n+2|n}{\equiv} \mathcal{O}^{S}_{ab}(\partial^{n-1}C,\ \partial^{n-k}S^{(k)},\ \partial^{n-k}R^{(k)})$$

Not altered if supplemented with an action that only adds to the omitted lower-order terms.

see Figueras, **Held**, Kovacs, 2407.08775 for a complete proof

... admits wave-like 2nd order field equations.

Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M^2_{\mathrm{Pl}}\, R \quad ✓$$

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\boxed{\alpha_0 R_{ab}R^{ab} - \beta_0\, R^2}\right] \quad ✓$$

$$\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M^2_{\mathrm{Pl}}}\Big[\boxed{\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R} + \boxed{\gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}} + \delta_{3,1}\, C_{abcd}C^{abcd}R + \delta_{3,2}\, C_{abcd}R^{ac}R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab}R^{ab}R + \delta_{3,5}\, R^3\Big] \quad ✓$$

Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M^4_{\mathrm{Pl}}}\Big[\boxed{\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R} + \boxed{\gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^*C^{abcd})^2} + \ldots\Big] \quad ✓$$

Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

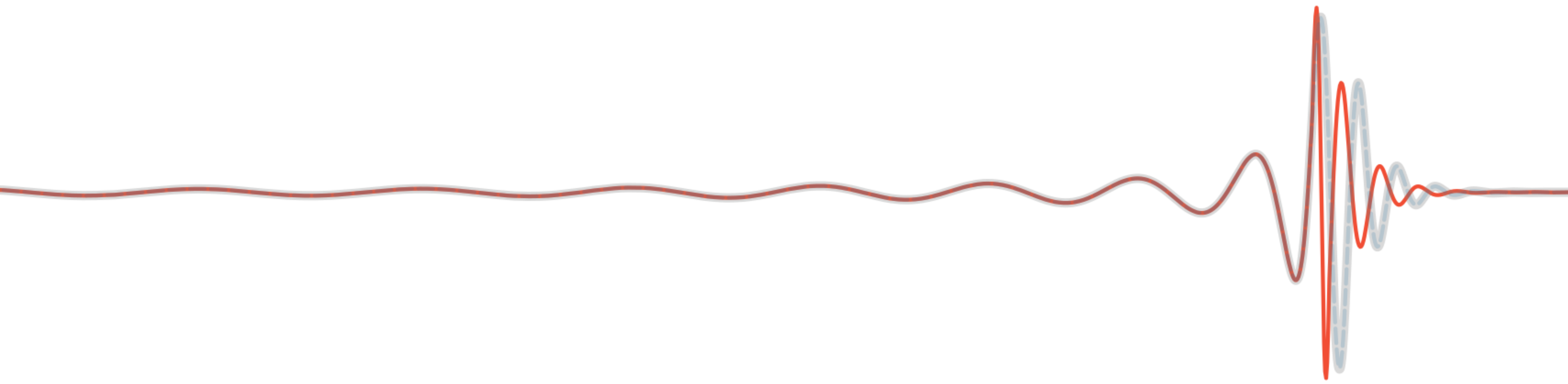
order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

Figueras, Held, Kovacs, 2407.08775

... of general effective field theories of gravity.

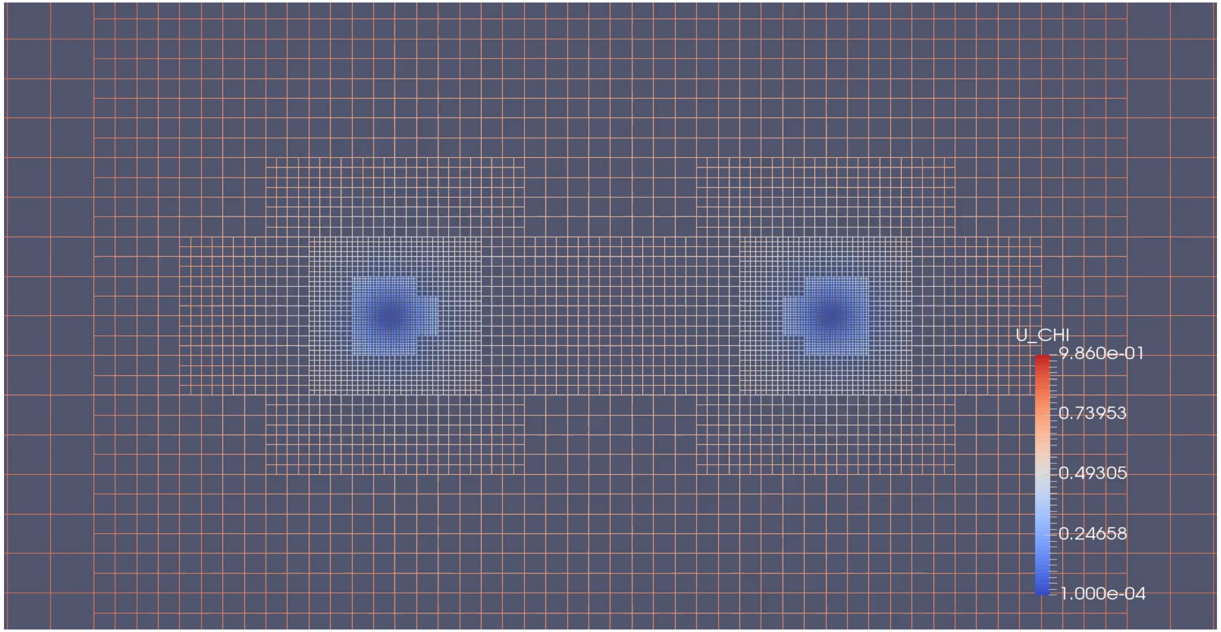


Part II: Nonlinear evolution & black hole binaries

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
Held, Lim, 2503.13428

High-performance computing …



- 4^{th} order finite differencing
- 4^{th} order Runge-Kutta timestepping
- adaptive mesh refinement
- parallelized on HPC clusters

Dendro-GR https://github.com/paralab/Dendro-GR

… required to solve the full (3+1) binary problem.

Black-hole binaries ...

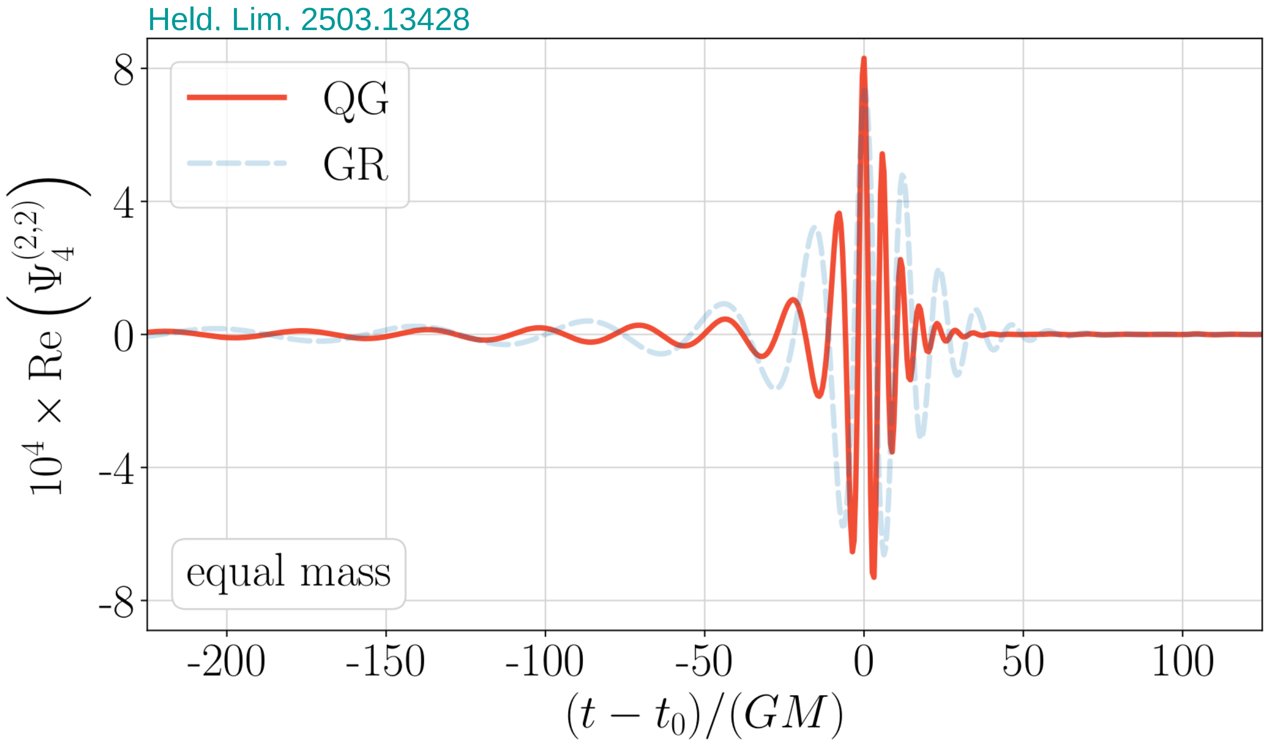
EFT regime of validity

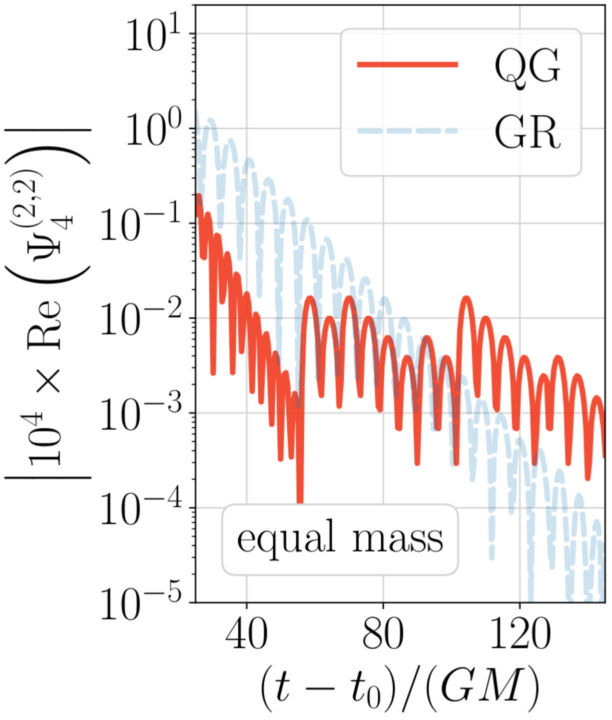
$GM\, m_2 \gg 1$	Held, Lim, PRD 108 (2023) 10 **no deviations**
$GM\, m_2 \sim 1$	**quantitative deviations**
$GM\, m_2 \lesssim 1$	**qualitative deviations** Held, Lim, 2503.13428

Waveforms for $GM\, m_2 \lesssim 0.43$ ...

Held, Lim, 2503.13428

QG masses		Binary parameters			
$G\, m_0\, M_2$	$G\, m_2\, M_2$	$\sqrt{G}\, M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.2	1	1	0	0



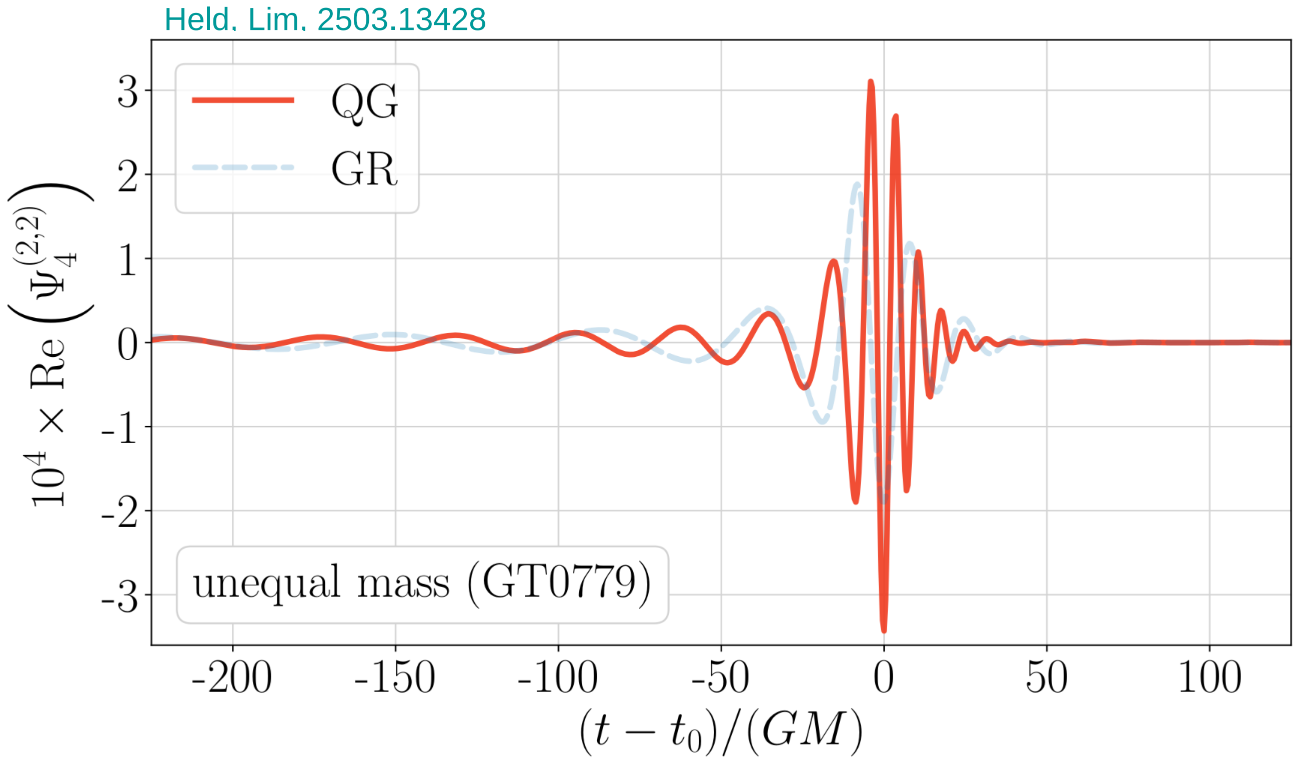


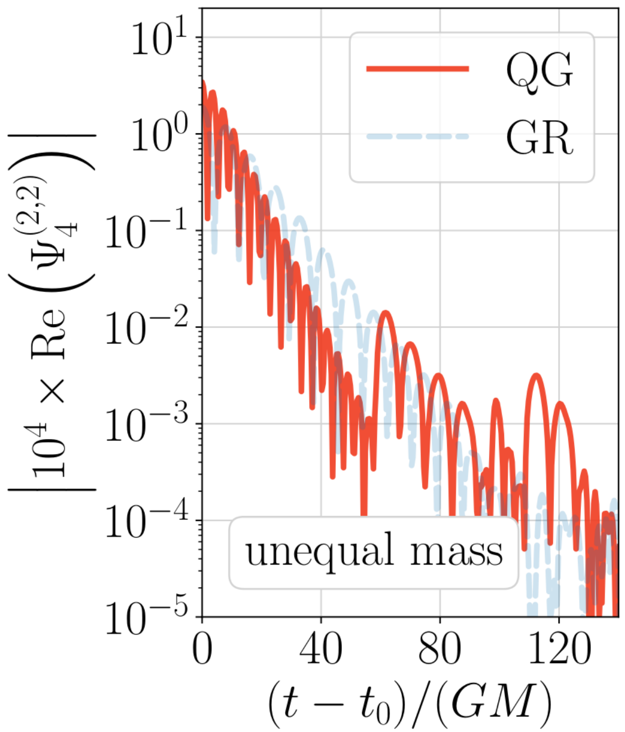
... deviate quantitatively.

Waveforms for $GM\,m_2 \lesssim 0.43$ ...

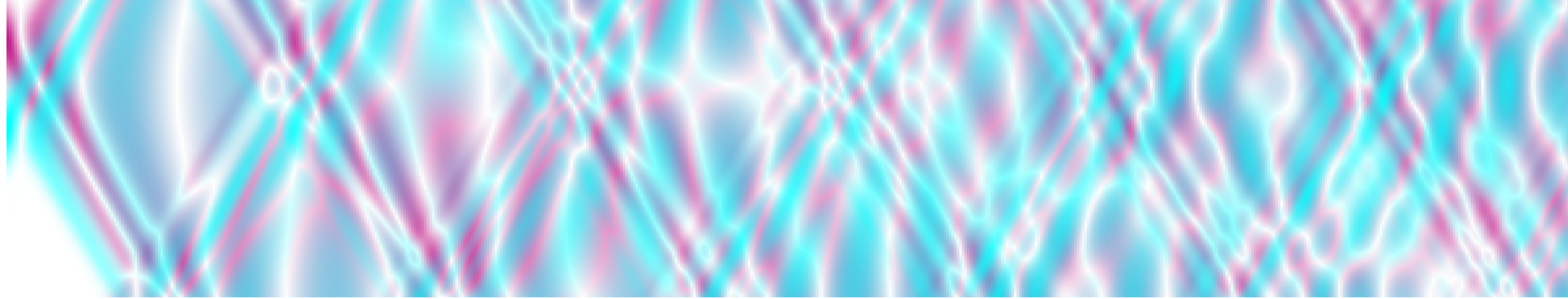
Held, Lim, 2503.13428

QG masses		Binary parameters			
$G\,m_0\,M_2$	$G\,m_2\,M_2$	$\sqrt{G}\,M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.2	1	5	-0.696	0





... deviate qualitatively.



Classical field theories with ghosts can be longlived

Propagating heavy ghosts achieves a well-posed IVP

Access to nonlinear time-evolution in the EFT of gravity