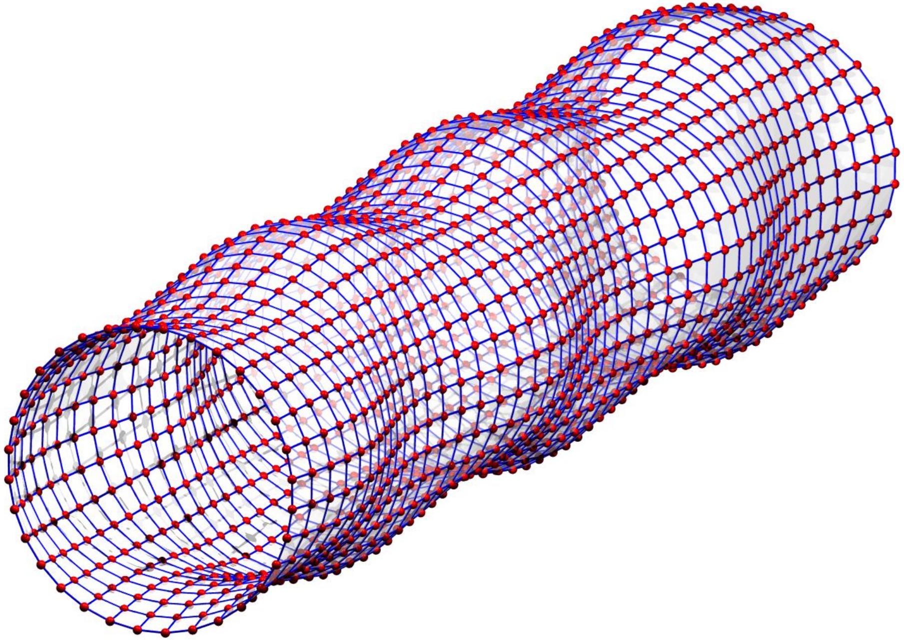
Project of the **LISA Cosmology Working Group**
Coordinators: Macarena Lagos, Alberto Mangiagli

Testing GW polarizations with LISA

Paola C. M. Delgado - CEICO - FZU



Beyond GR

Lovelock's theorem: GR propagates with two degrees of freedom

Modified Gravity:

Scalar-tensor
Vector-tensor
Lorentz-breaking
…

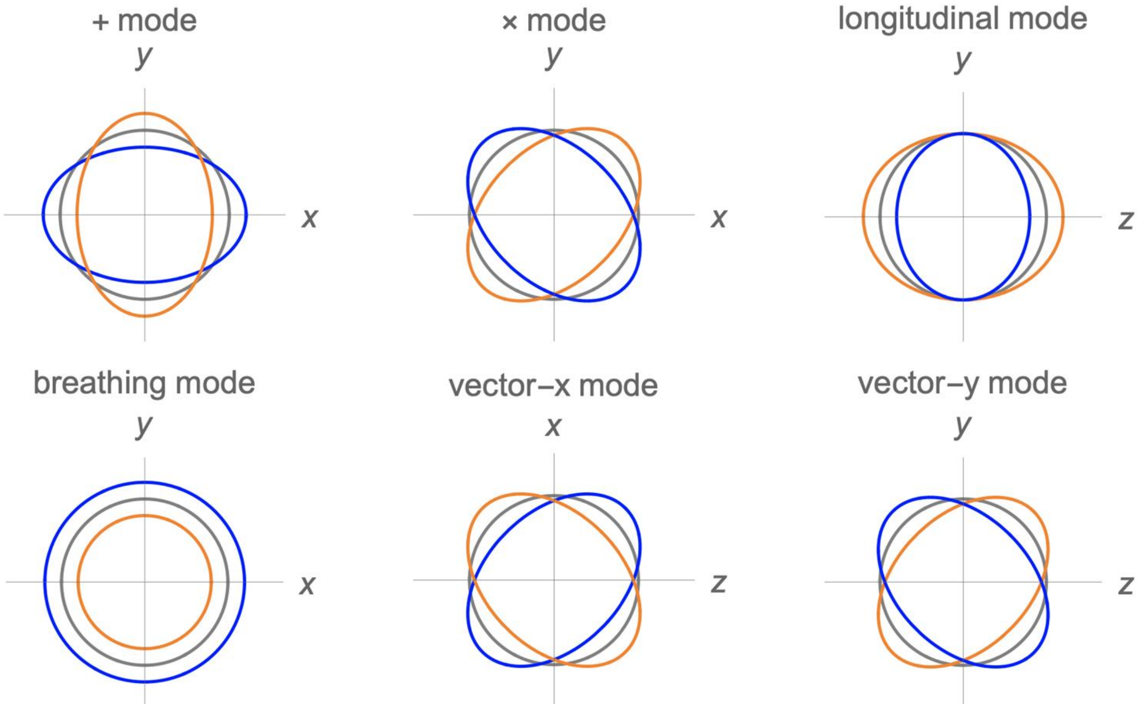
→

Extra degrees of freedom

→

Changes in tensor modes
+
Extra polarizations

GW Polarizations



$$h_{ij} = \begin{pmatrix} h_b + h_+ & h_\times & h_{v1} \\ h_\times & h_b - h_+ & h_{v2} \\ h_{v1} & h_{v2} & h_l \end{pmatrix}$$

Theory-agnostic Approach

Goals:

- Develop parametrized model for extra polarizations (ppE formalism);
- Use specific gravity theories as inspiration:
  - Emission: modified binary evolution + extra polarizations;
  - Propagation: assume all polarizations propagate at the same constant speed.
- Forecast precision on extra polarizations;
- Map parametrized model to known gravity theories.

Theory-agnostic Approach

Multipolar Decomposition

$$h_+(t) - i h_\times(t) = \sum_{\ell,m} h_T^{(\ell,m)}(t) Y_{-2}^{(\ell,m)}(\iota, \phi_c)$$

$$h_{v_1}(t) - i h_{v_2}(t) = \sum_{\ell,m} h_V^{(\ell,m)}(t) Y_{-1}^{(\ell,m)}(\iota, \phi_c)$$

$$h_{b,l}(t) = \Re \sum_{\ell,m} h_{b,l}^{(\ell,m)}(t) Y^{(\ell,m)}(\iota, \phi_c)$$

Spin-weighted spherical harmonics

ppE Approach

PN expansion For ℓ=1,2
[Chatziinoannou+ 2012]

T = Tensor , p = {V, b, l}

Dominant harmonic

$$\tilde{h}_T^{(2,2)}(f) = A^{(2)} \left(1 + \alpha u^a\right) e^{-i\Psi_{GR}^{(2)}} e^{i\beta u^b} \,,$$

$$\tilde{h}_p^{(2,2)}(f) = A^{(2)} \alpha_{p2} u_2^{a_p} e^{-i\Psi_{GR}^{(2)}} e^{i2\beta_1 u_2^b} \,,$$

$$\tilde{h}_p^{(1,1)}(f) = A^{(1)} \alpha_{p1} u_1^{a_p} e^{-i\Psi_{GR}^{(1)}} e^{i\beta_1 u_1^b} \,,$$

(K. Schumacher, Y. Xie,+)

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(K. Schumacher, Y. Xie,+)

Connection between ppE and specific theories

For the Tensor part:

Theories	PPE Phase Parameters: Magnitude (β)	PPE Phase Parameters: Exp. (b)	Binary Type
Scalar-Tensor [95, 96]	$-\frac{5}{7168}\eta^{2/5}(\alpha_1-\alpha_2)^2$	-7	Any
EdGB [97]	$-\frac{5}{7168}\zeta_{\rm EdGB}\frac{\left(m_1^2\tilde{s}_2^{\rm EdGB}-m_2^2\tilde{s}_1^{\rm EdGB}\right)^2}{m^4\eta^{18/5}}$	-7	Any
DCS [82, 98]	$\frac{481525}{3670016}\eta^{-14/5}\zeta_{\rm dCS}\left[-2\delta_m\chi_a\chi_s+\left(1-\frac{4992\eta}{19261}\right)\chi_a^2+\left(1-\frac{72052\eta}{19261}\right)\chi_s^2\right]$	-1	BH/BH
Einstein-Æther [99]	$-\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\rm EA}-s_2^{\rm EA})^2}{[(1-s_1^{\rm EA})(1-s_2^{\rm EA})]^{4/3}}\left[\frac{(c_{14}-2)w_0^3-w_1^3}{c_{14}w_0^3w_1^3}\right]$	-7	Any
Khronometric [99]	$-\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\rm kh}-s_2^{\rm kh})^2}{[(1-s_1^{\rm kh})(1-s_2^{\rm kh})]^{4/3}}\sqrt{\bar{\alpha}_{\rm kh}}\left[\frac{(\bar{\beta}_{\rm kh}-1)(2+\bar{\beta}_{\rm kh}+3\bar{\lambda}_{\rm kh})}{(\bar{\alpha}_{\rm kh}-2)(\bar{\beta}_{\rm kh}+\bar{\lambda}_{\rm kh})}\right]^{3/2}$	-7	Any
Noncommutative [100]	$-\frac{75}{256}\eta^{-4/5}(2\eta-1)\Lambda^2$	-1	BH/BH
Varying-G [92]	$-\frac{25}{851968}\eta_0^{3/5}\dot{G}_{C,0}\left[11m_0+3(s_{1,0}+s_{2,0}-\delta_{\dot{G}})m_0-41(m_{1,0}s_{1,0}+m_{2,0}s_{2,0})\right]$	-13	Any

[Tahura&Yagi 2019]

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[Tahura&Yagi 2019]

Connection between ppE and specific theories

Work in progress for extra polarizations:

Theories	a	b	a_b	a_l	a_{v_1}	a_{v_2}
Scalar-Tensor [47]	$-2, 0$	-7,-5	0	-6	-	-
Einstein-Æther [48]	0	-5	0	0	0	0
Rosen's theory [28]	0	-7,-5	0	0	0	0
Lightman-Lee Theory [28]	0	-7,-5	0	0	0	0
Lorentz-Breaking [cite]	?	?	?	?	?	?

(A. Garoffolo, M. Zhu, S. Akama, M. Lagos, J. Zosso, L. Perivolaropoulos, A. Nilsson,+)

+ expressions for the mapping to $\alpha_{p1,p2}$ and $\beta_{1,2}$ for the extra polarizations.

The mapping to ppE formalism for extra polarizations is a new result of this project

How many (extra) parameters to infer?

GR

11 parameters

- 2 masses
- 1 distance
- 2 spin magnitudes
- 2 angles (sky position)
- 2 angles (inclination & polarization)
- Coalescence phase & time

ppE for Tensor

2 parameters

- α, β

$$\tilde{h}_T^{(2,2)}(f) = \tilde{h}_{\rm GR}^{(2,2)}(f)(1+\alpha u^a)e^{i\beta u^b}$$

ppE for scalar & vector

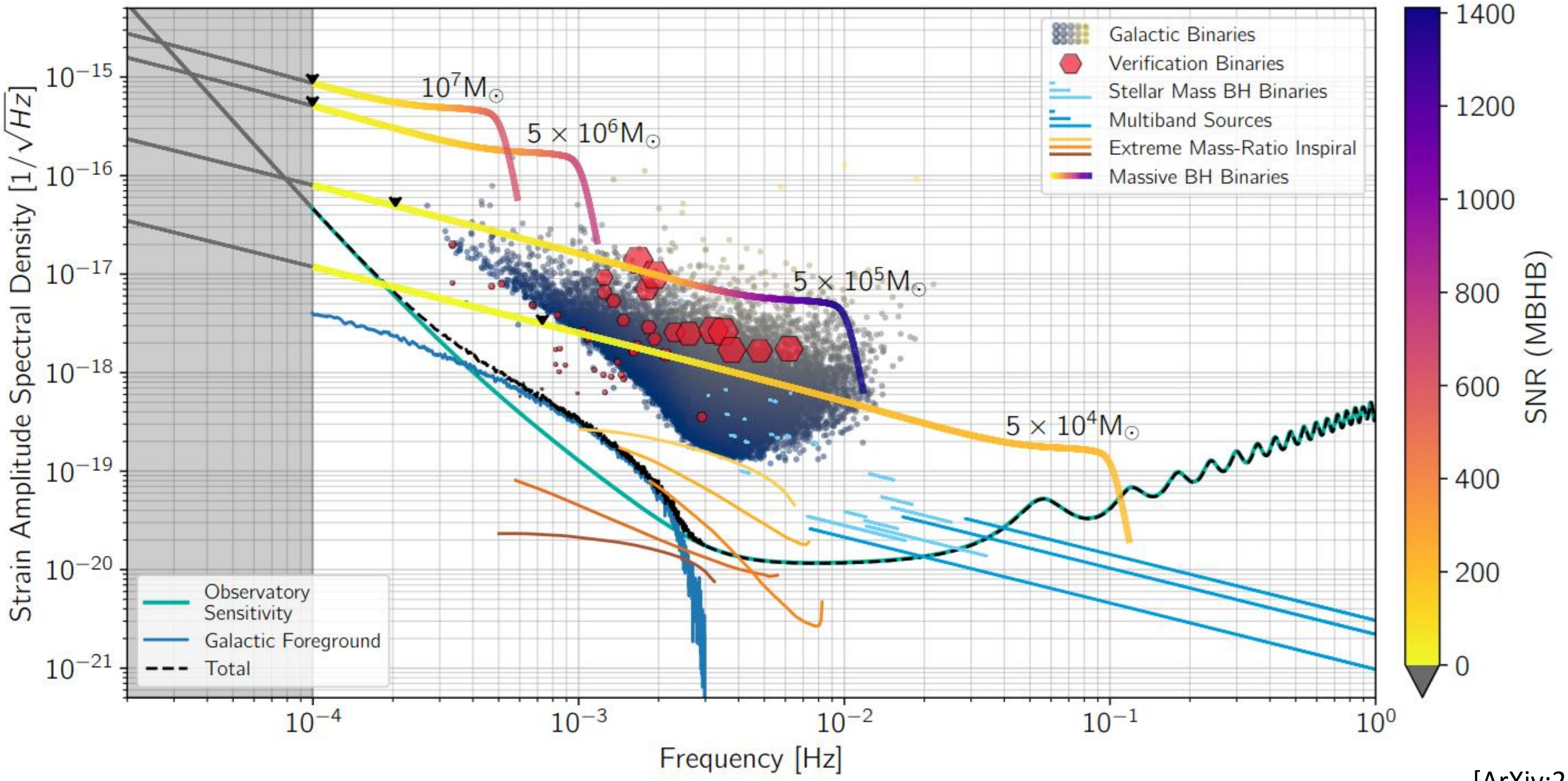
9 parameters

- α_p^j, β_1

p={V,l,b}

j={(1,1),(2,2)}

Which sources are we targeting?



[ArXiv:2402.07571]

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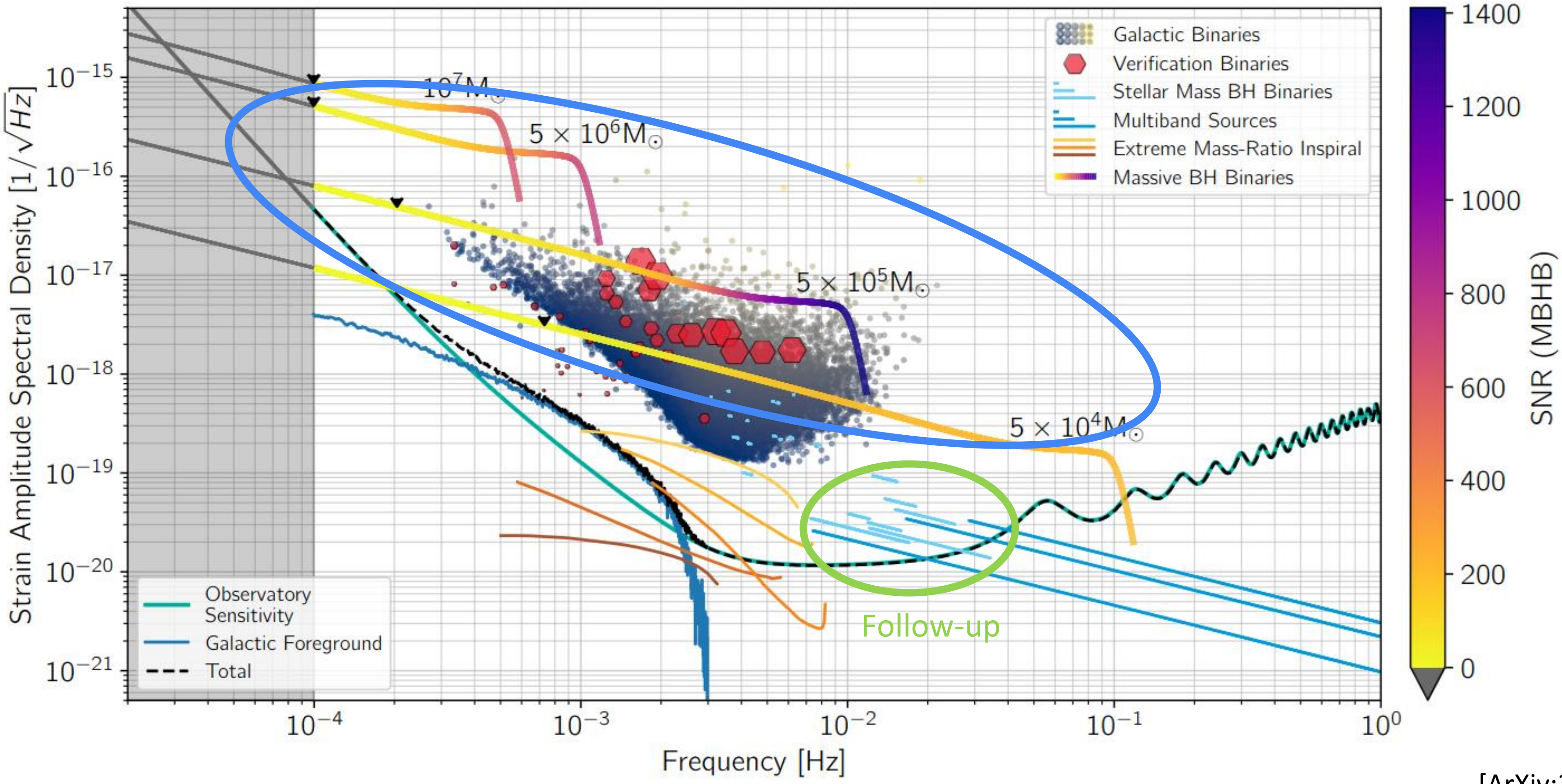
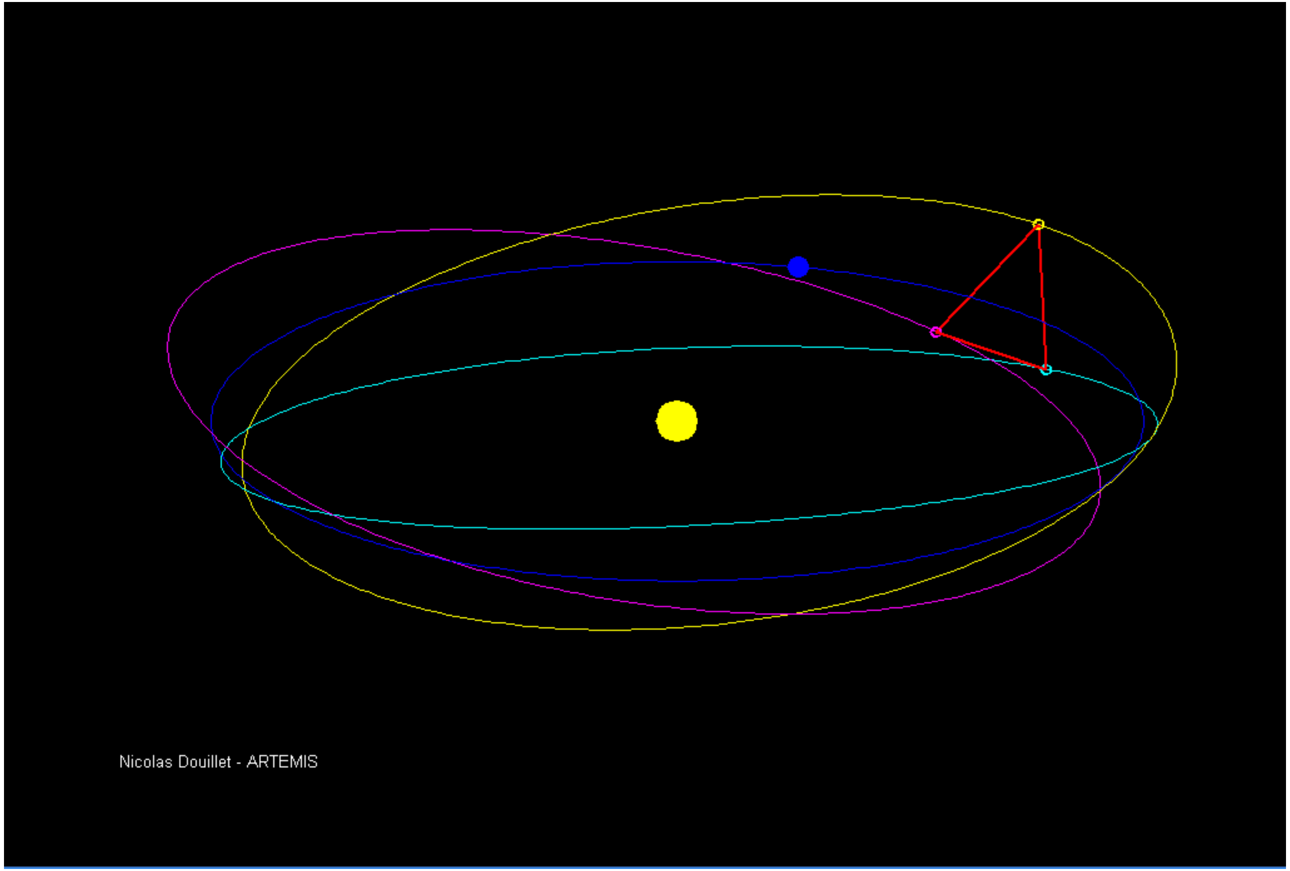


[ArXiv:2402.07571]

Why LISA?



- High SNR: up to 10^3;
- Long inspirals: weeks to years (time-dependent modulations from the detector's motion);
- Probe 3D GW distortions;
- Wavelength comparable to arm length: optimal for scalar polarizations.

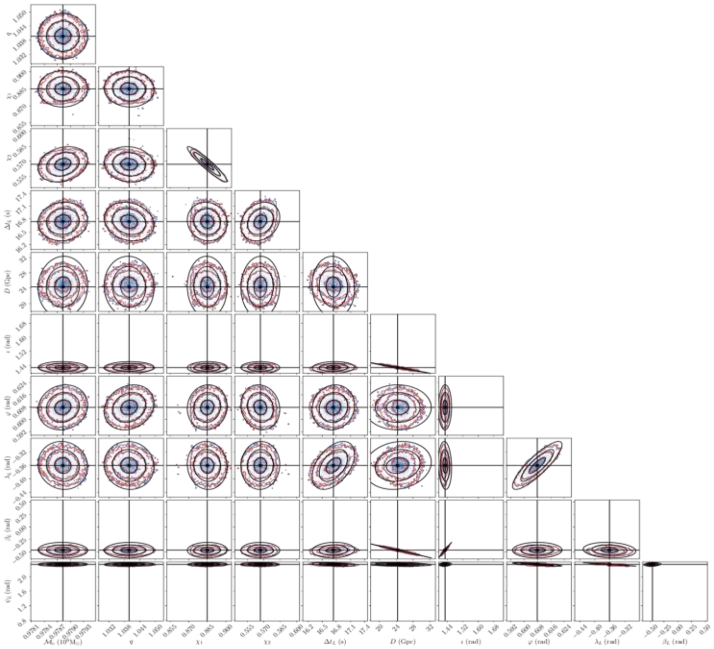
Simulate the GW signal

We use *lisabeta* (Marsat+20) to simulate the GW signal from SBHBs and MBHBs.

https://gitlab.in2p3.fr/marsat/lisabeta_release

✓ IMRPhenomXHM
✓ Include low frequency response (motion of the detector) + high frequency response
✓ Fisher+Bayesian analysis
✓ Repository stored on gitlab.in2p3 → easily accessible to members of the project
✓ $h_{+/\times}$ implemented via spherical harmonics → extension to extra-polarisations

(M. Corman, A. Mangiagli)



Inspiral-only or Inspiral-Merger-Ringdown?

For MBHBs, we have two further options: to focus on inspiral-only analysis or to include merger and ringdown.

Inspiral-only

✓ Non-GR modification is cut when it's still well understood

❌ Worse estimates on the binary parameters

Inspiral-Merger-Ringdown

✓ Higher harmonics help breaking degeneracies

❌ Need to suppress the extra-polarisations with a window function: spurious effect might be introduced

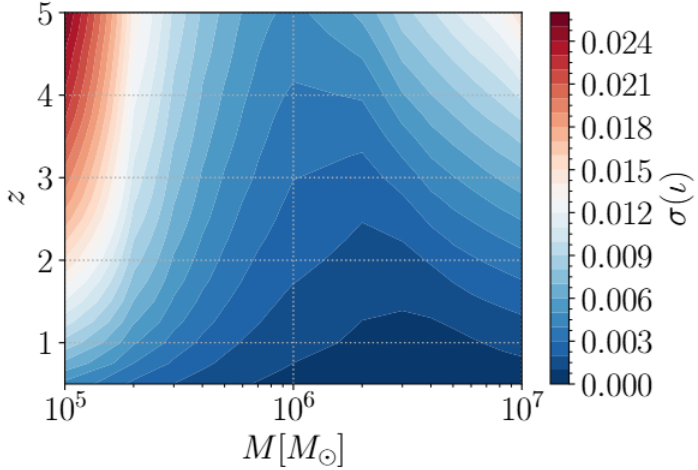
Current status: Inspiral-only - done.
Inspiral-Merger-Ringdown - running phase.

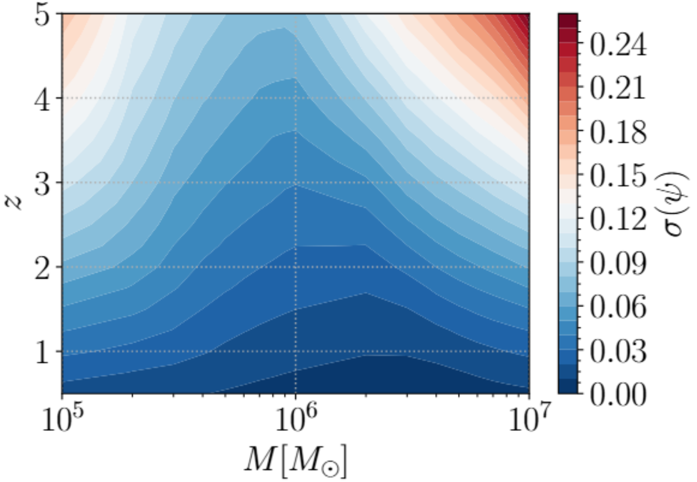
GR as a benchmark

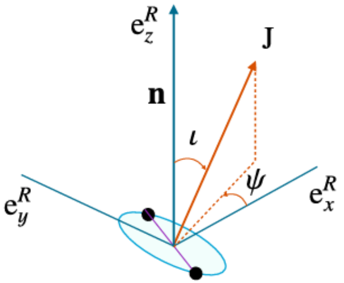
We checked LISA ability to constrain inclination and polarization angles for MBHBs in GR:

(P. C. M. Delgado, G. Orlando, R. Theriault)

Polarization angles: inclination $\iota \in [0,\pi]$ and polarization $\psi \in [0,\pi]$







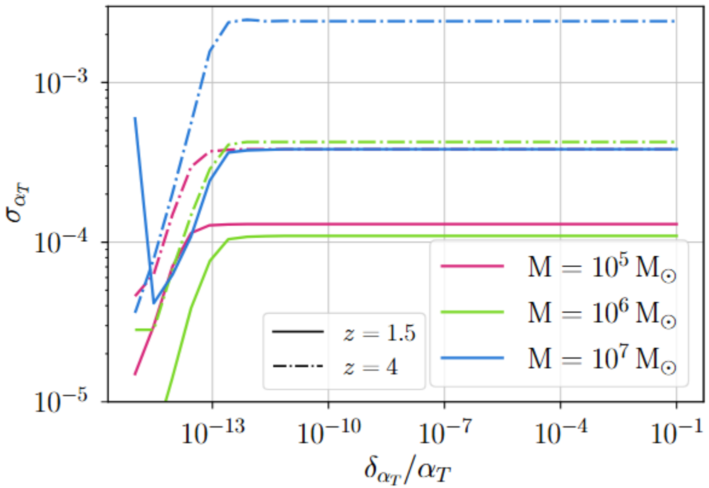
[arXiv:1409.2349]

Results confirmed by few MCMC runs.

(M. Corman)

Fisher steps for the ppE parameters

Fisher steps (fractional, non-fractional, all ppE parameters): (P. C. M. Delgado, A. Mangiagli)

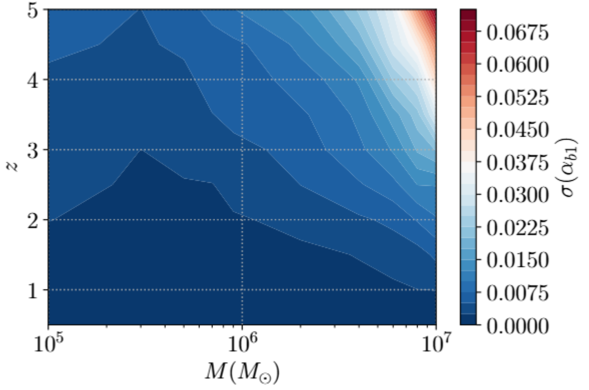


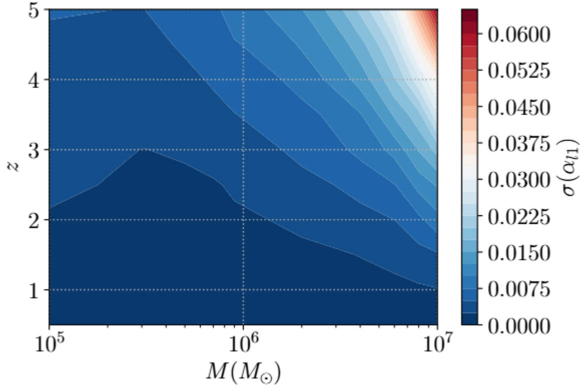
GR injections – inspiral-only

We include ppE parameters for extra polarizations in the Fisher parameters and infer the errors:

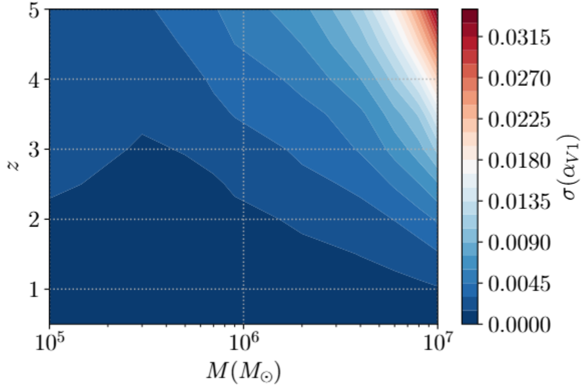
(P. C. M. Delgado)

(1,1) modes:



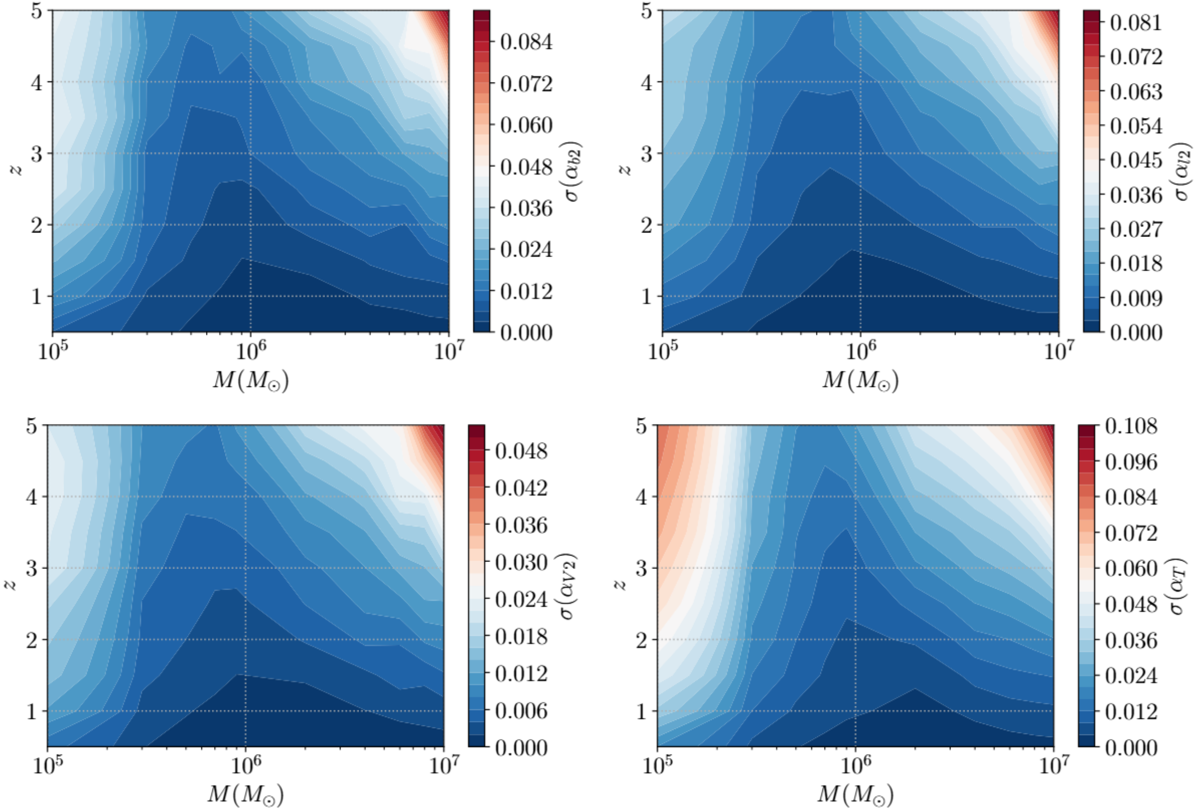


> "If GR is correct and the signal contains only the usual tensor modes, how tightly could we *rule out* or *constrain* the presence of extra polarizations?"



GR injections – inspiral-only

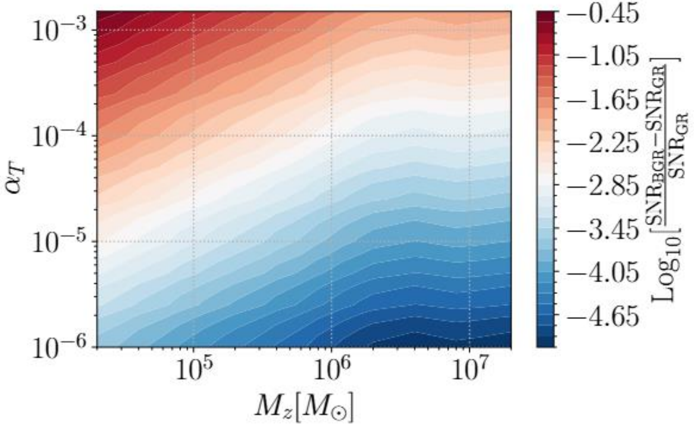
(2,2) modes:



Preliminary results – Inspiral-only

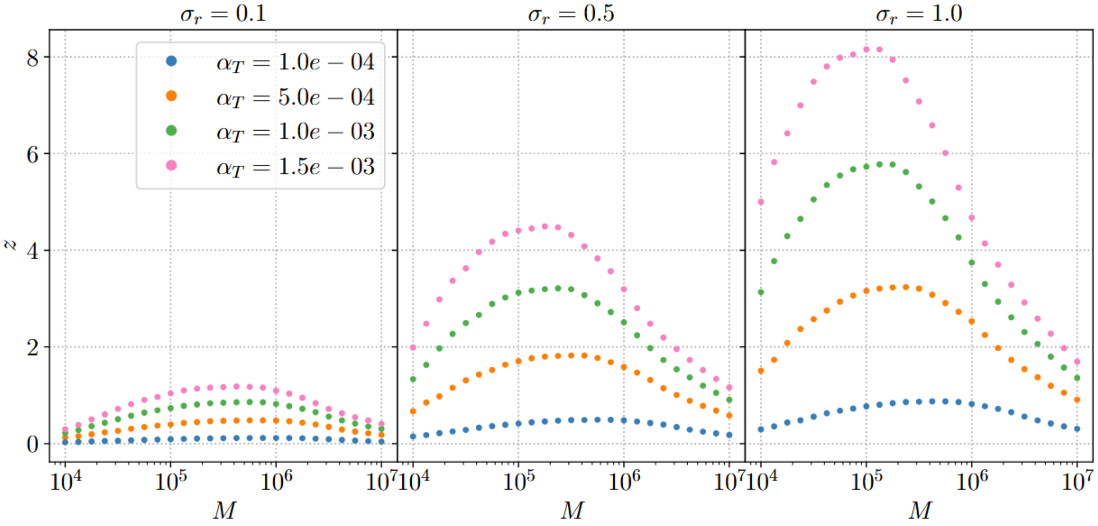
(M. Corman, P. C. M. Delgado, A. Mangiagli)

SNR variation due to the presence of the ppE parameters:



Tensor mode:

Relative errors on the ppE parameters:



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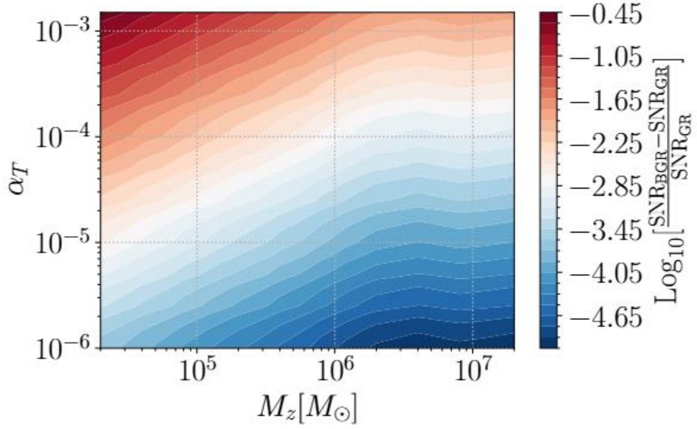
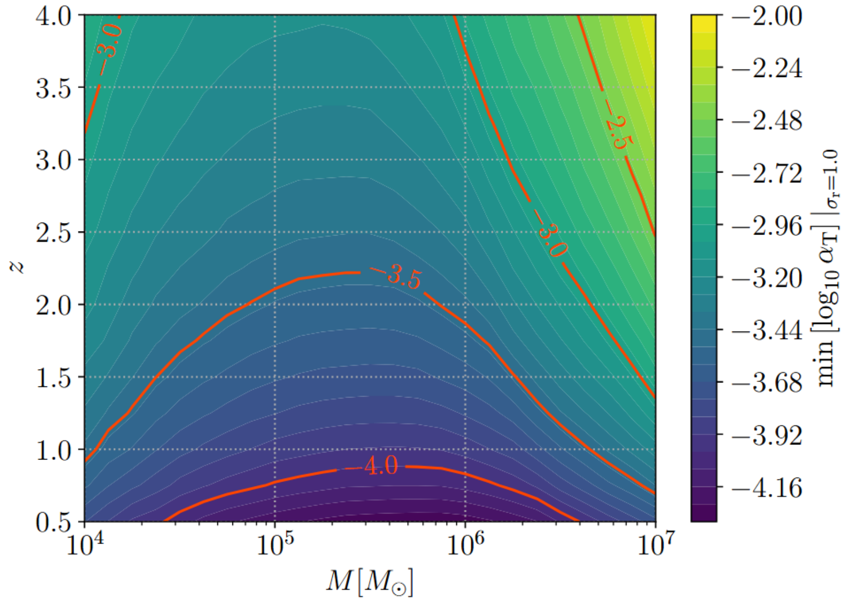


Minimum value of the ppE parameter for a relative error of 1.0:



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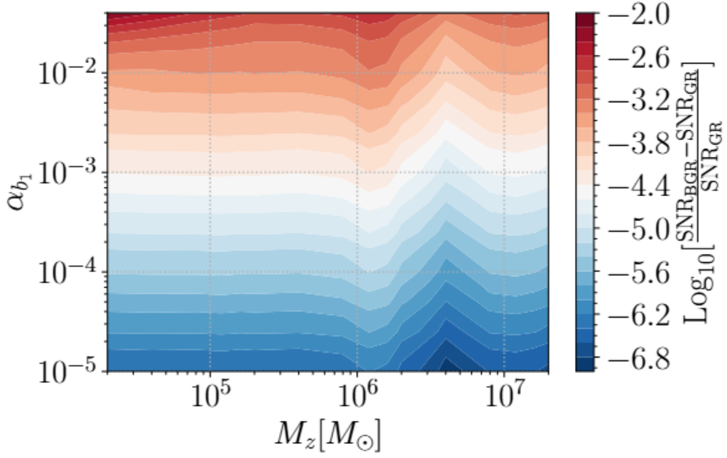
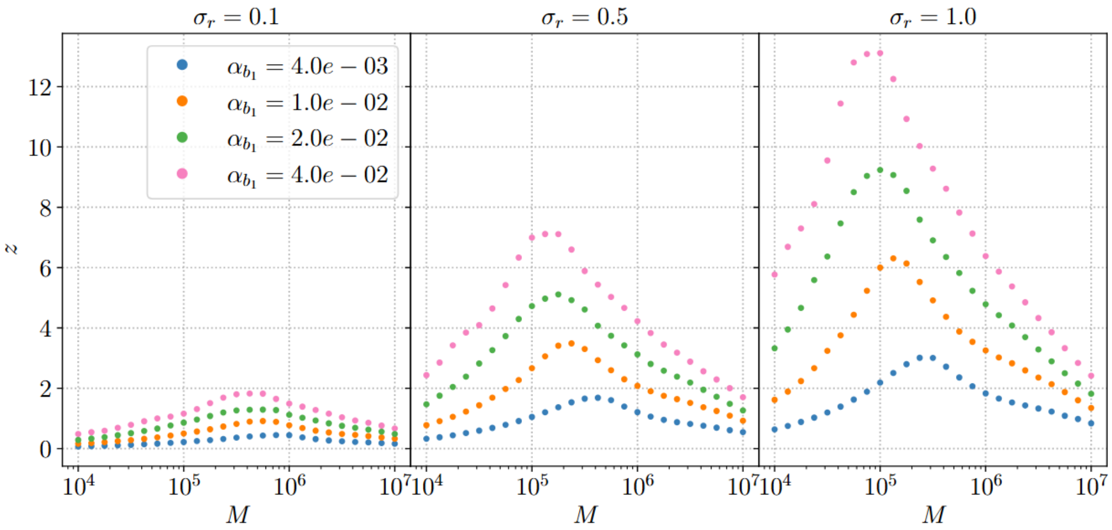


Breathing mode (dipole):

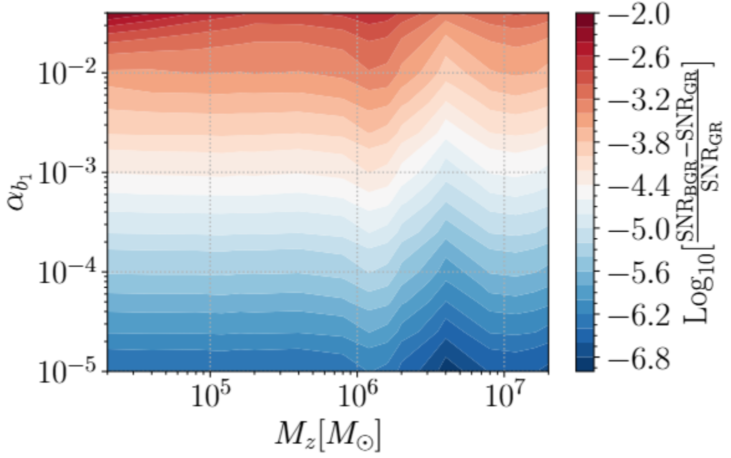
Relative errors on the ppE parameters:



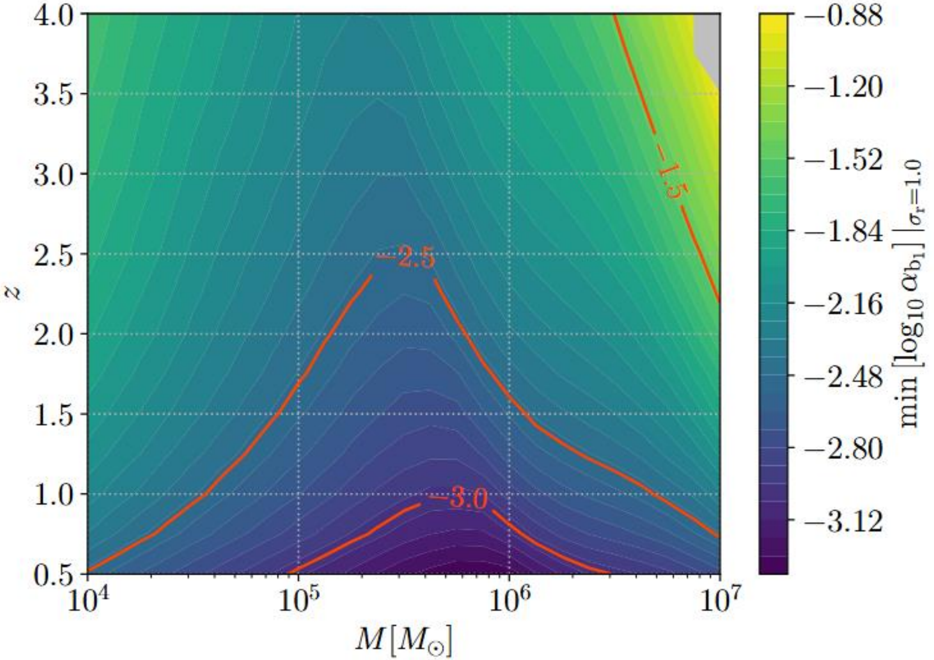
Preliminary results – Inspiral-only

(M. Corman, P. C. M. Delgado, A. Mangiagli)

SNR variation due to the presence of the ppE parameters:



Minimum value of the ppE parameter for a relative error of 1.0:



Preliminary results – Inspiral-only

(M. Corman, P. C. M. Delgado, A. Mangiagli)

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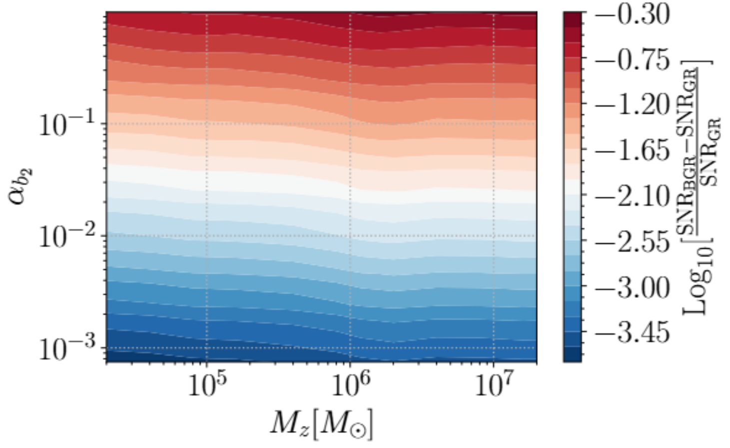
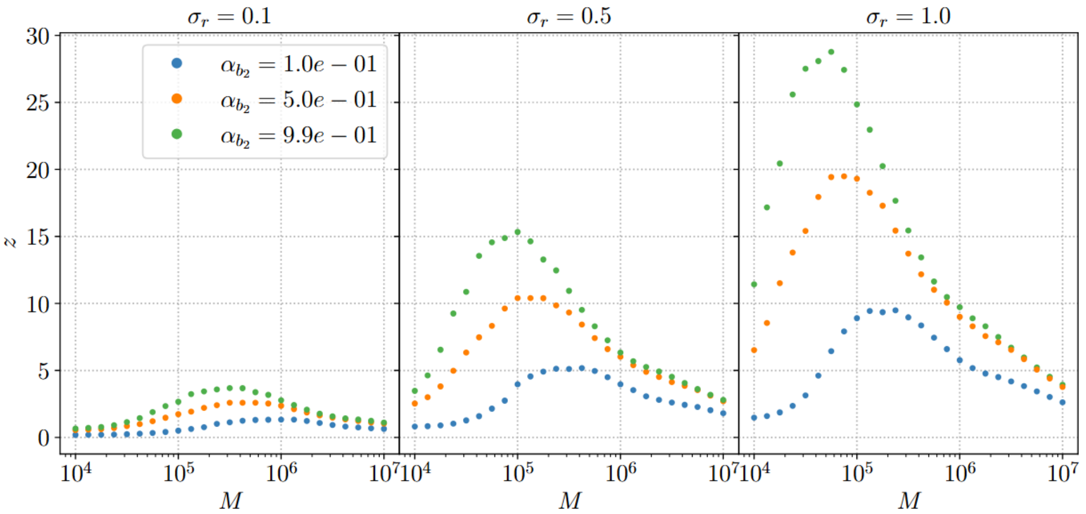


Breathing mode (quadrupole):

Relative errors on the ppE parameters:



Preliminary results – Inspiral-only

(M. Corman, P. C. M. Delgado, A. Mangiagli)

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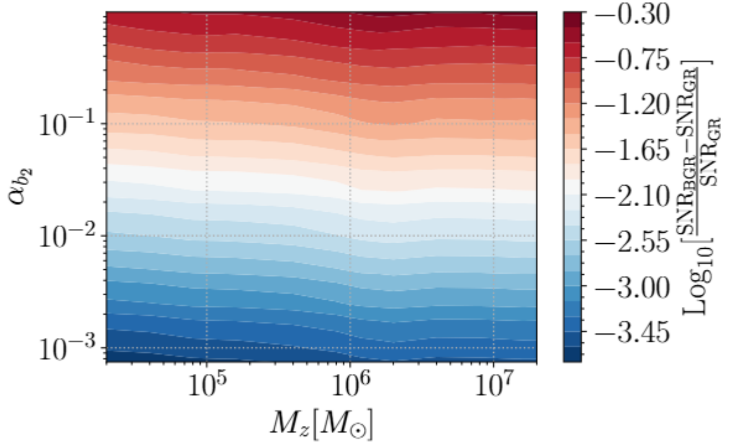

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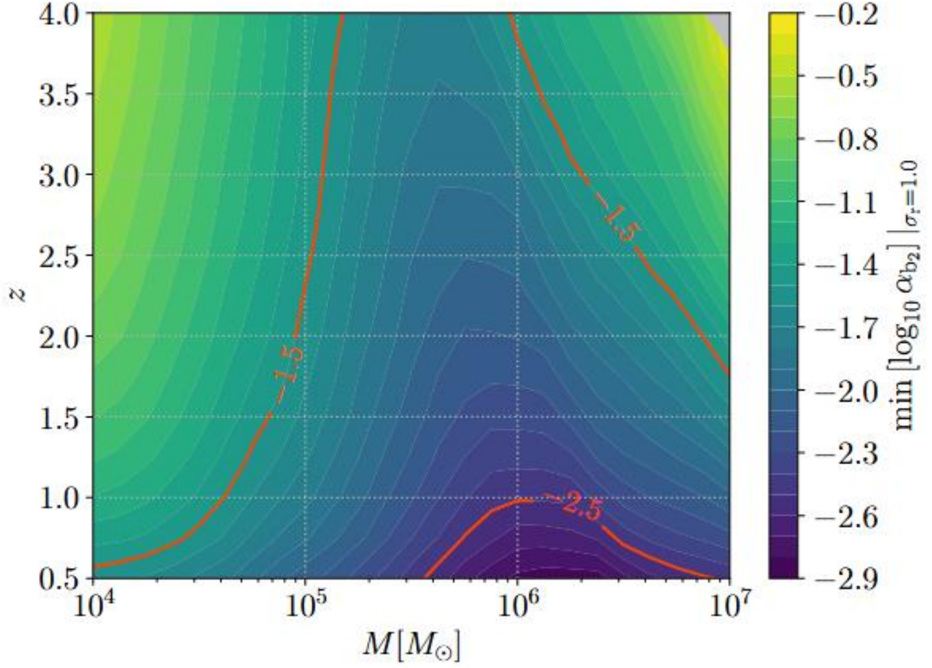
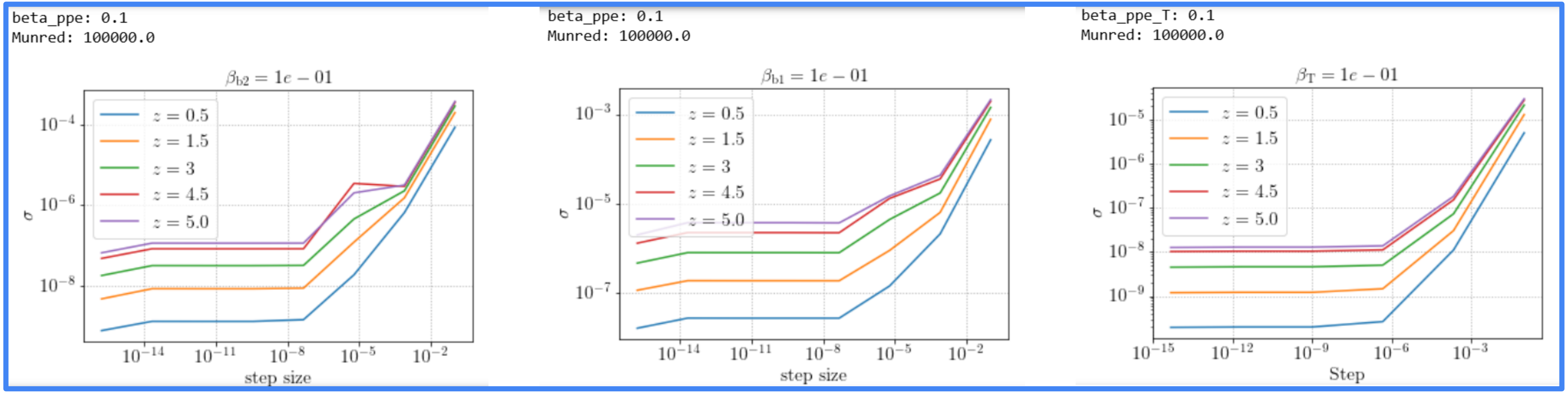


Preliminary results

For the phase modifications we have recently implemented the phase alignment (rerunning results). (M. Corman, M. Piarulli, A. Mangiagli, S. Marsat)

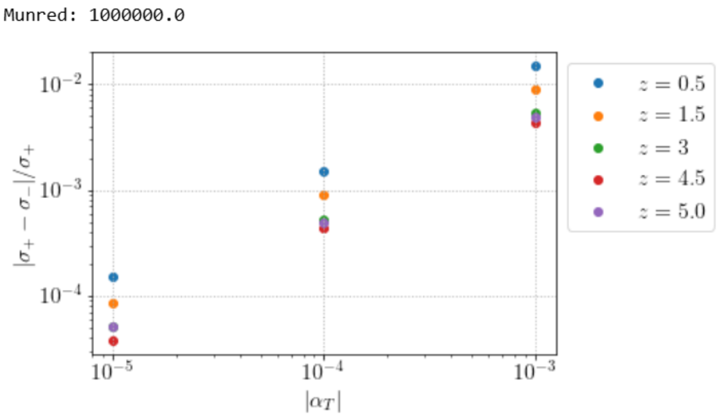
Preliminary results regarding the orders of magnitude: (P. C. M. Delgado, M. Corman, A. Mangiagli)

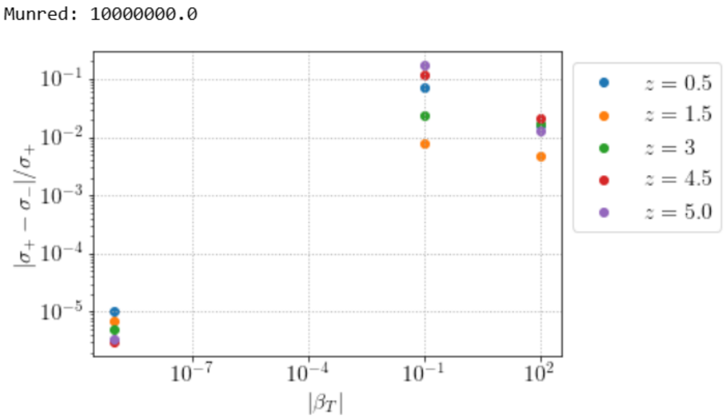
Including alignment, merger & ringdown:

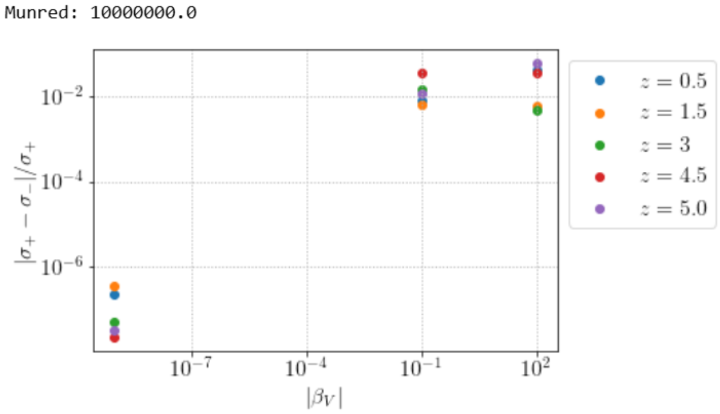


Positivity of ppE parameters (α_T, β_T, β_P)

(P. C. M. Delgado, A. Mangiagli)







The error differences between positive and negative values of the ppE parameters (when possible) are very small.

The analysis for non-individual systems is being currently made.

Massive BHBs and Stellar BHBs

Massive BHBs

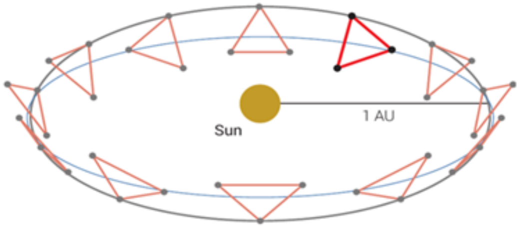
✓ Strong SNR

✓ Inspiral-merger-ringdown

❌ Short signal (max ~1 month)

❌ Not favoured in some theories

$h_{EdGB} \sim 1/m_{tot}^4$



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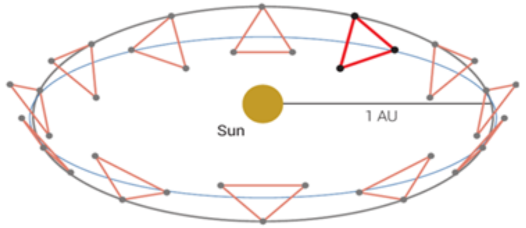


Stellar BHBs

✓ Long inspiral (~years)
✓ Excellent determination of extrinsic parameters
✓ Wavelength comparable to arm length: optimal for scalar polarizations [Tinto+ 2010]
❌ Low SNR

Concluding remarks

- LISA will be able to test extra polarizations that appear in modified theories of gravity (both amplitude and phase modifications);
- Our project approaches the forecasts in a theory-independent way considering the ppE parametrization of the waveforms;
- The mapping to specific theories allows us to relate the constraints on the ppE parameters to constraints on the theory parameters;
- The mapping of extra polarisations is a new result of this collaborative project.

Follow-up projects:

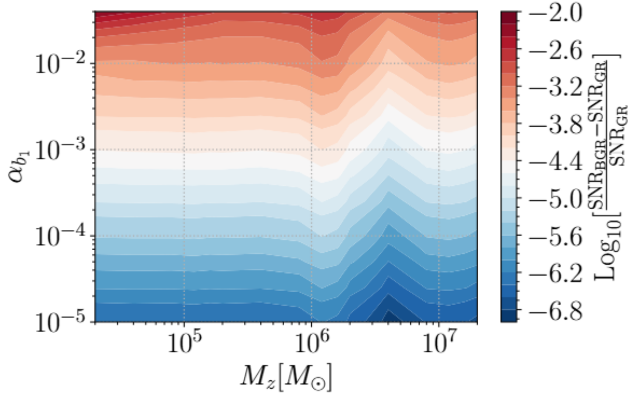
- SBHB constraints (data analysis sub-group);
- MCMC analysis (data analysis sub-group);
- Theories derivations and ppE mapping; (theory sub-group)

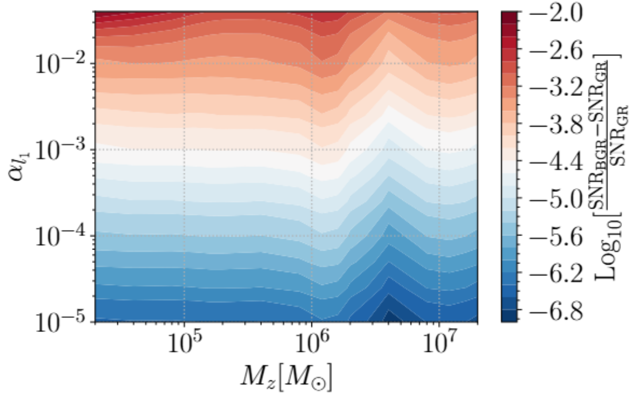
Thank you!

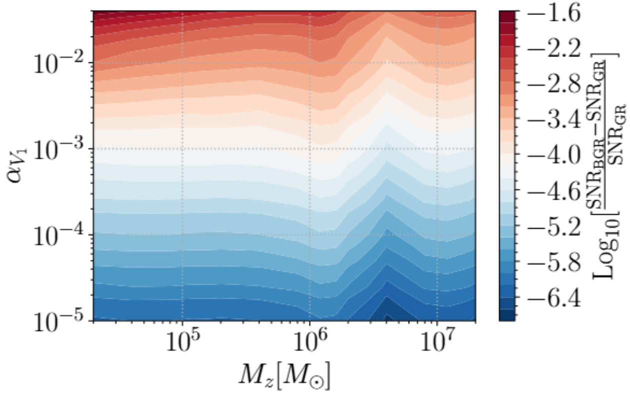
Backup slides

Preliminary results – Inspiral-only

(M. Corman, P. C. M. Delgado, A. Mangiagli)

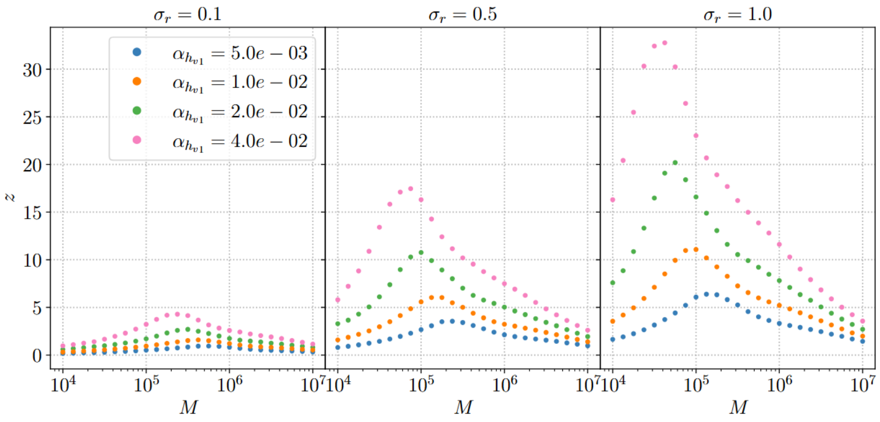


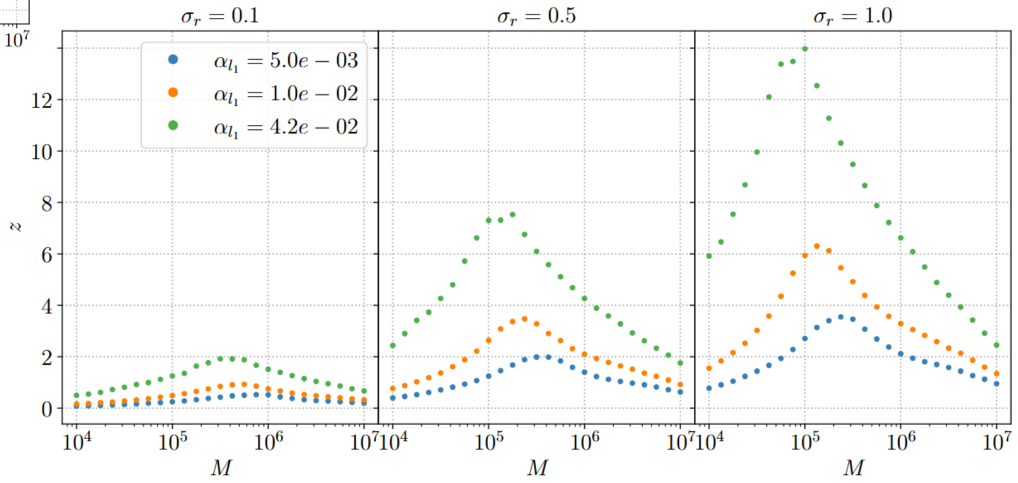




Preliminary results – Inspiral-only

(M. Corman, P. C. M. Delgado, A. Mangiagli)





Preliminary results – Inspiral-only

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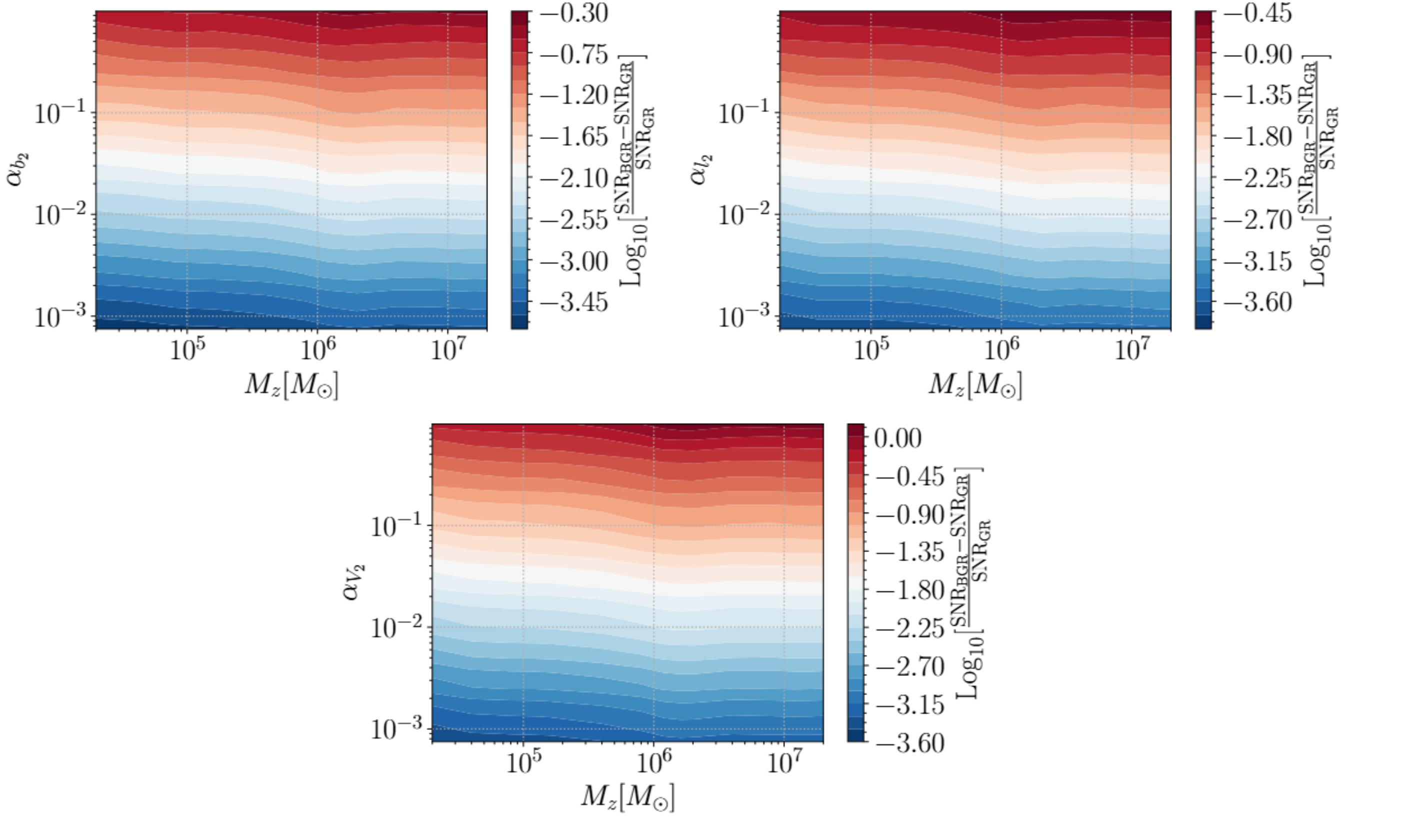

Preliminary results – Inspiral-only

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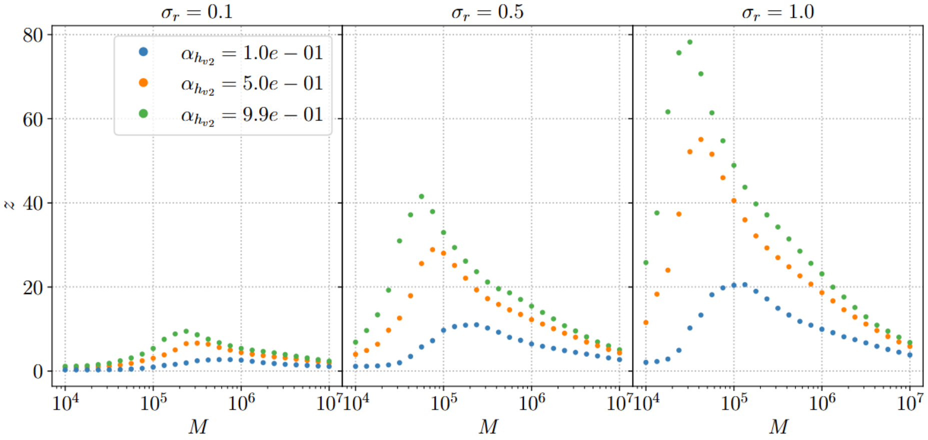

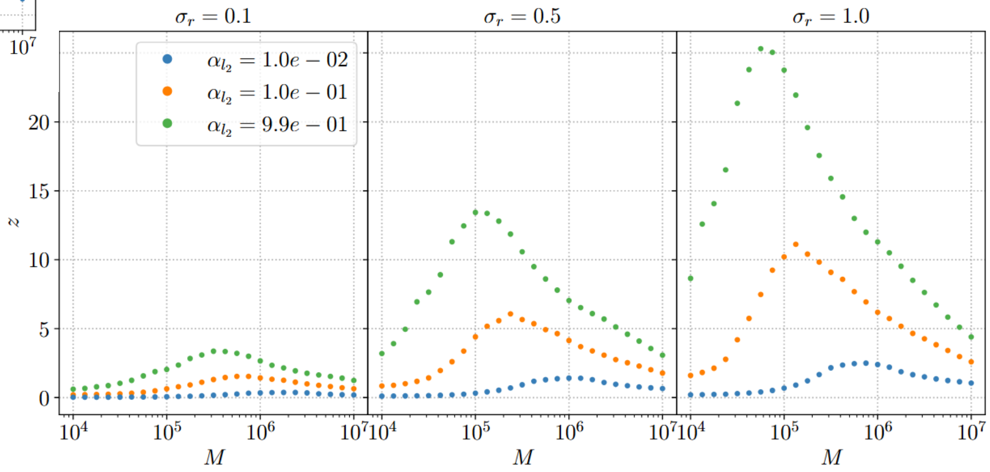

Comparison with Flexible Theory Independent (FTI) approach

$$\tilde{h}(2,2)(f) = A_{22} e^{-i(\psi_{22}^{GR} + \delta\psi_{22})},$$

$$\tilde{h}(2,2)(f) = A_{22} e^{-i(\psi_{22}^{GR} - \beta u^{b})}$$

$$\delta\psi_{22} = \frac{3}{128\bar{q}\nu^5}\left[\sum_{n=-2}^{7} \delta\psi_n \nu^n + \log \text{ terms}\right]$$

→ $b = -7$ → $-1\text{PN term } (n = -2)$

$\nu = (\pi f M)^{1/3}$ and $\bar{q} = q/(1+q)^2$

$$\delta\psi_{22} = \frac{3}{128\bar{q}\nu^7}\delta\psi_{-2}.$$

$$\beta = -\frac{3}{128\bar{q}}\left(\frac{M}{\mathcal{M}_z}\right)^{-7/3}\delta\psi_{-2}$$