Cherenkov vs. ghosts

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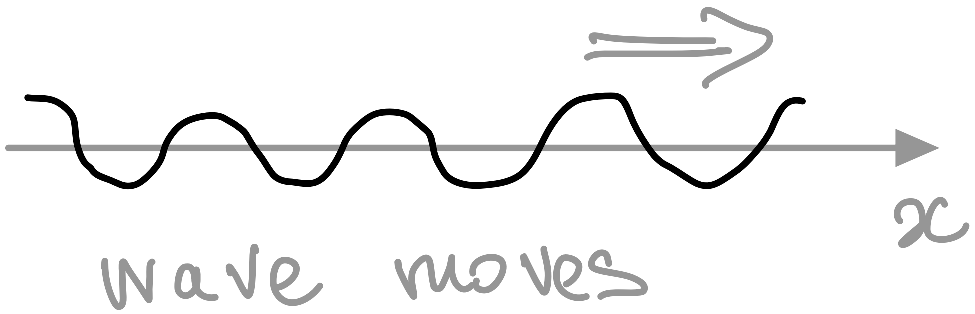
Outline

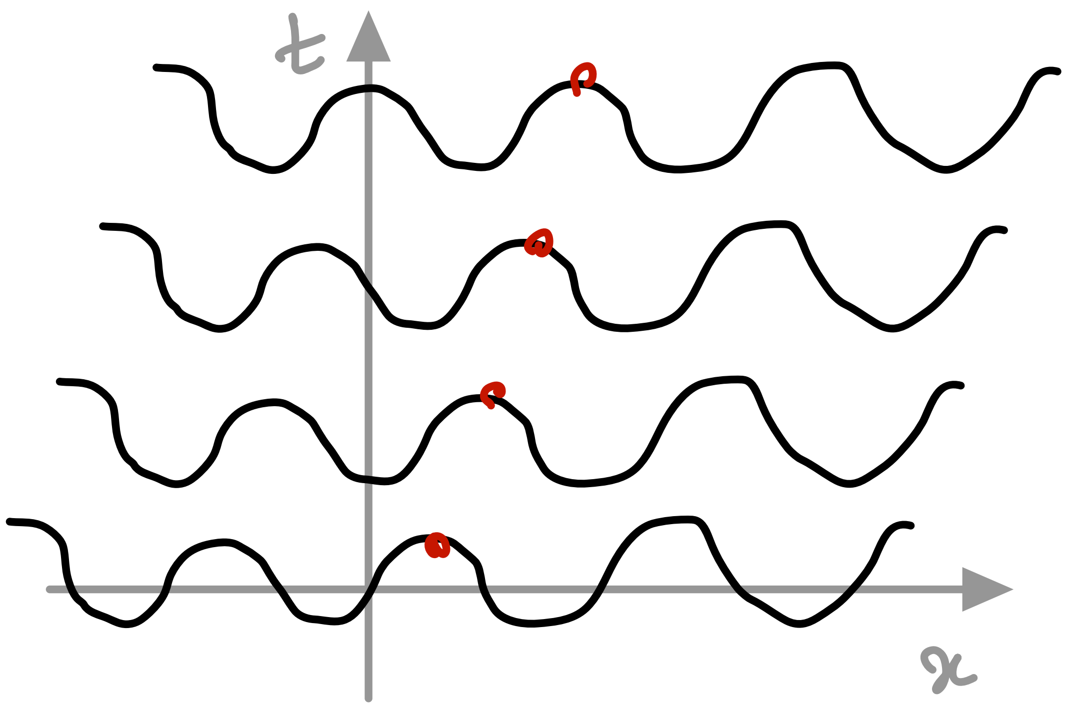
- **Cherenkov radiation**
- **Ghosts in modified gravity**
- **Cherenkov vs ghosts**

Signal propagation and dispersion relation



$\Box\varphi = 0 \quad \Rightarrow \quad \varphi = Ae^{-i\omega t + kx}$

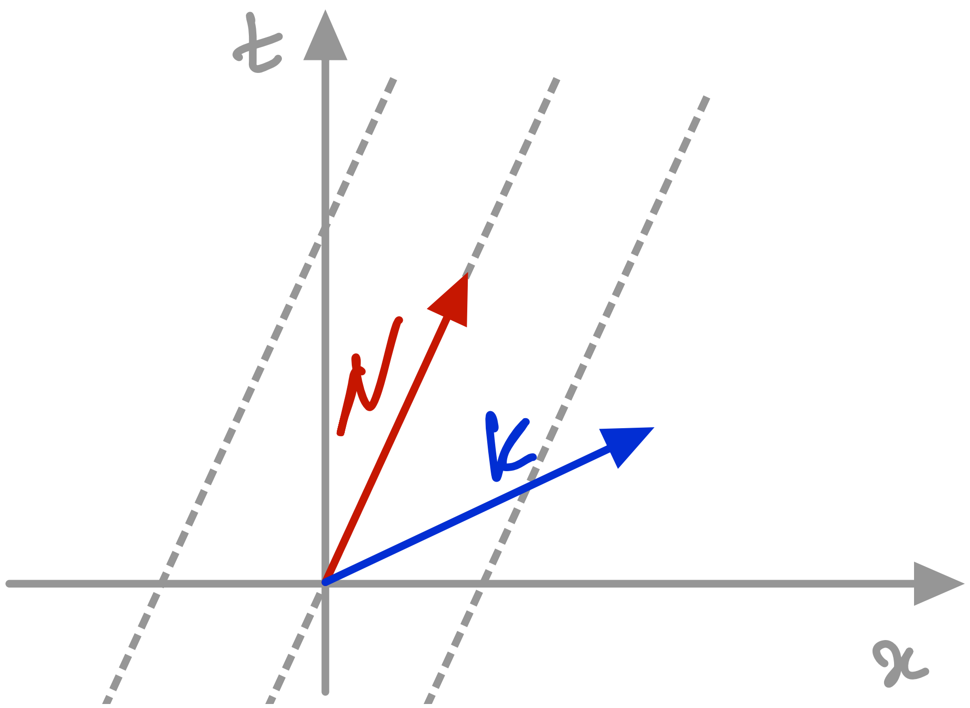
$\omega = c_s k \quad (c_s < 1)$



phase: $\Phi = -\omega k + kx$

propagation is along $\Phi =$ const (Also characteristics)

Signal propagation and dispersion relation



- $k_\mu = \nabla_\mu \Phi = \{-\omega, k\} \;\rightarrow$

  $k^\mu = \{\omega, k\}$

  k_μ is orthogonal to Φ =const

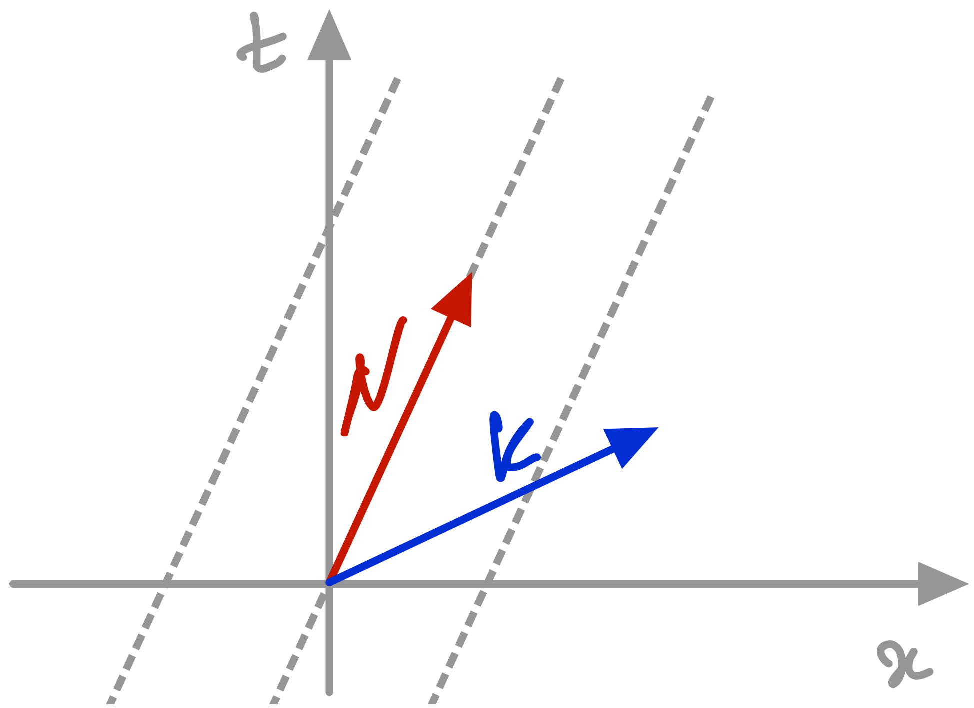
- Introduce N^μ : $N^\mu k_\mu = 0$

  N^μ is tangential to $\Phi =$ const i.e. N^μ is propagation vector

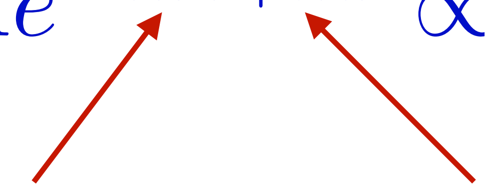
  $N^\mu = \{k, \omega\}$

- k^μ and N^μ are different

Signal propagation and dispersion relation



$$\varphi = Ae^{-i\omega t + kx} \propto e^{ik^\mu x_\mu}$$

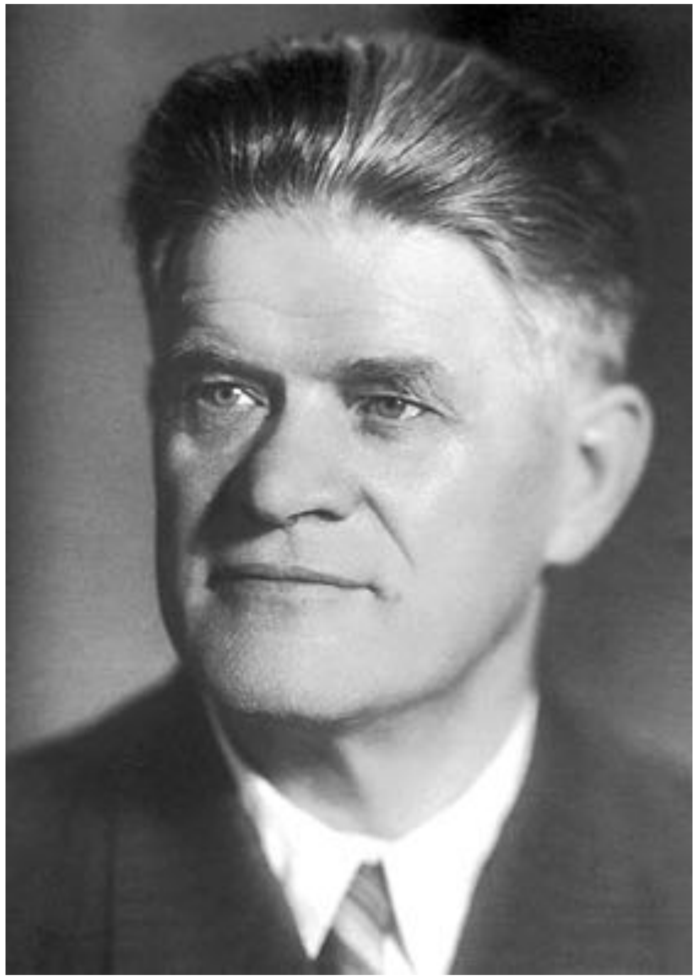


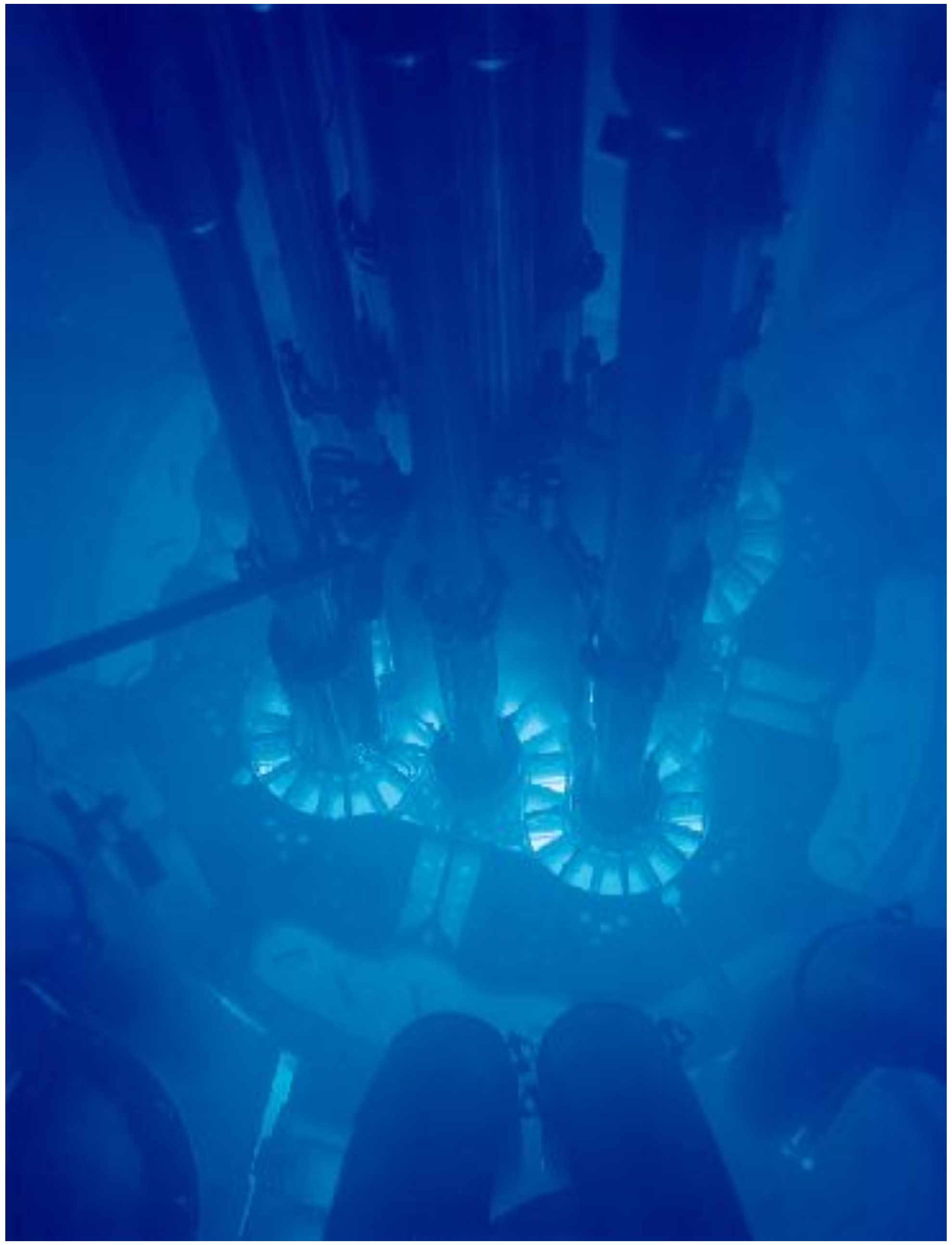
k^μ is 4-momentum (p^μ)

$$k^\mu = \{\omega, k\}, \quad \omega = c_s k$$

- subluminal case $\omega < k$ (for dust $\omega = 0$)
- null case $\omega = k$
- superluminal ease $\omega > k$

Cherenkov radiation





Electromagnetic radiation emitted when a charged particle passes through a medium at a speed greater than the velocity of light in that medium

Cherenkov radiation

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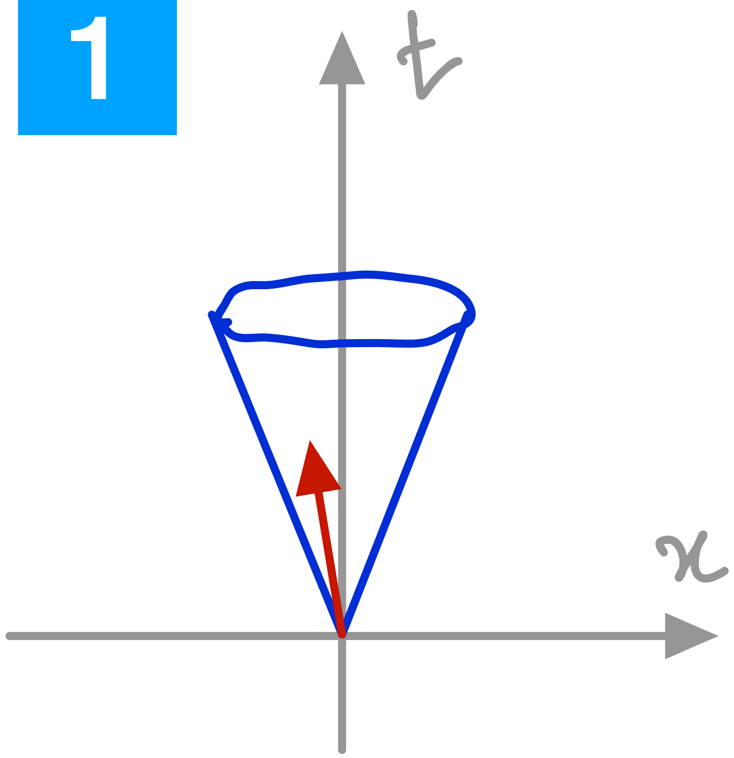
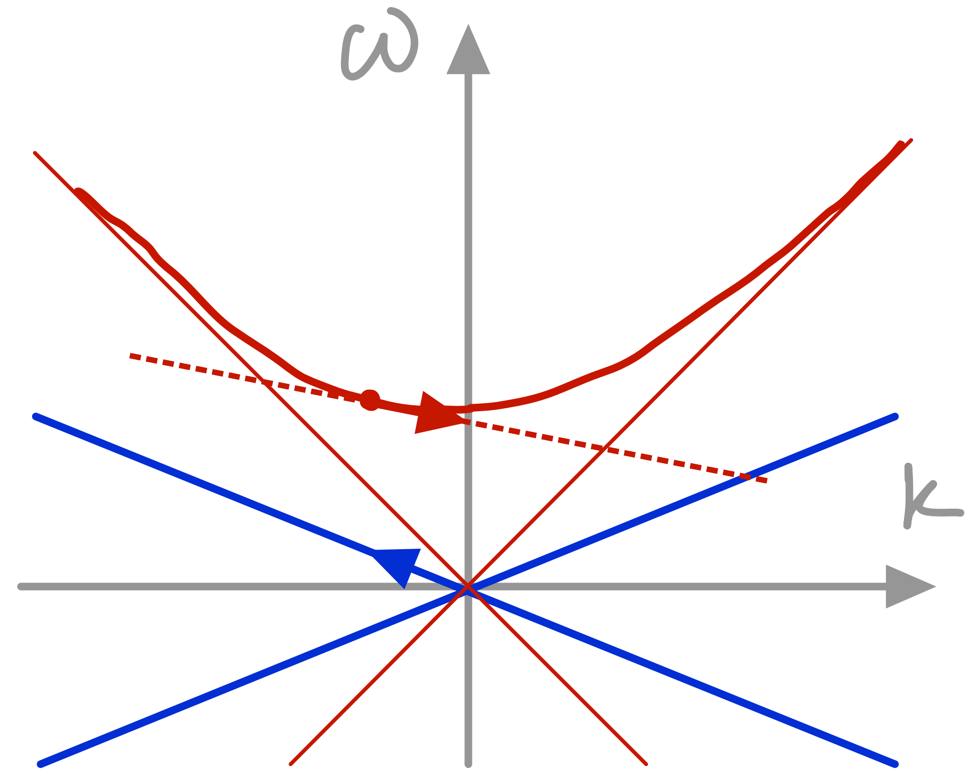




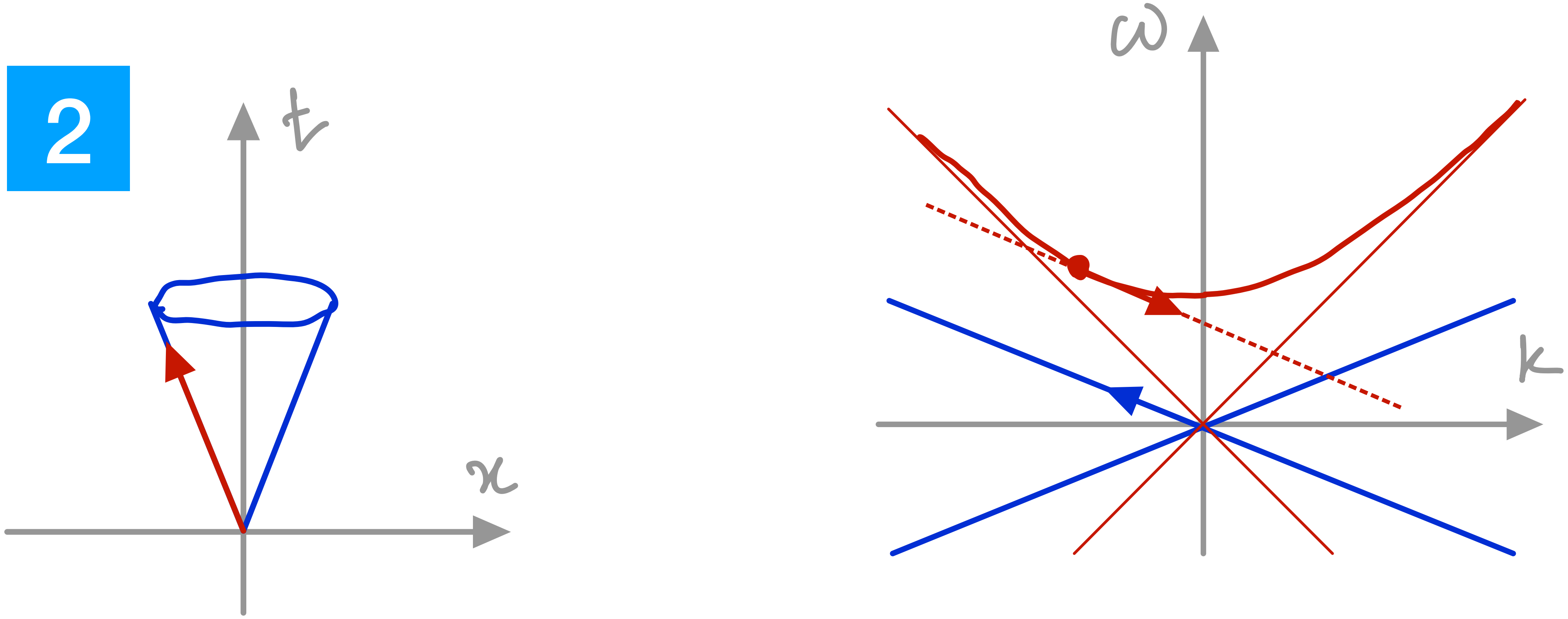
phonons: $k^\mu = \{c_s k, -k\}$

particle: $p^\mu = \left\{\sqrt{m^2 + k^2}, -k\right\}$

Can the particle loose its energy to give it to a phonon?

In this case no: $p_2^\mu \neq p_1^\mu + k^\mu$

Cherenkov radiation



How about now? Yes, the particle looses its energy to phonons.

Assuming interaction: $p_2^\mu = p_1^\mu + k^\mu$

Cherenkov Radiation (1+1)

Cherenkov radiation

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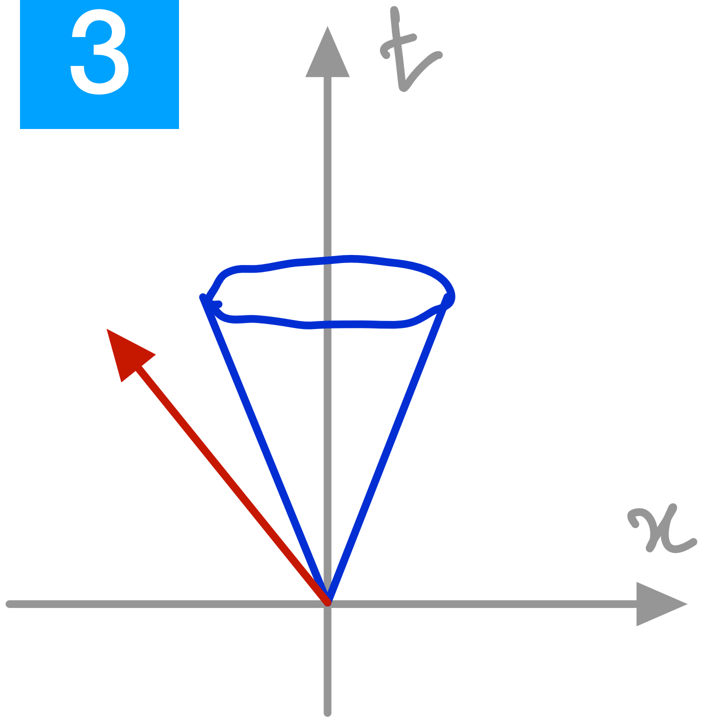
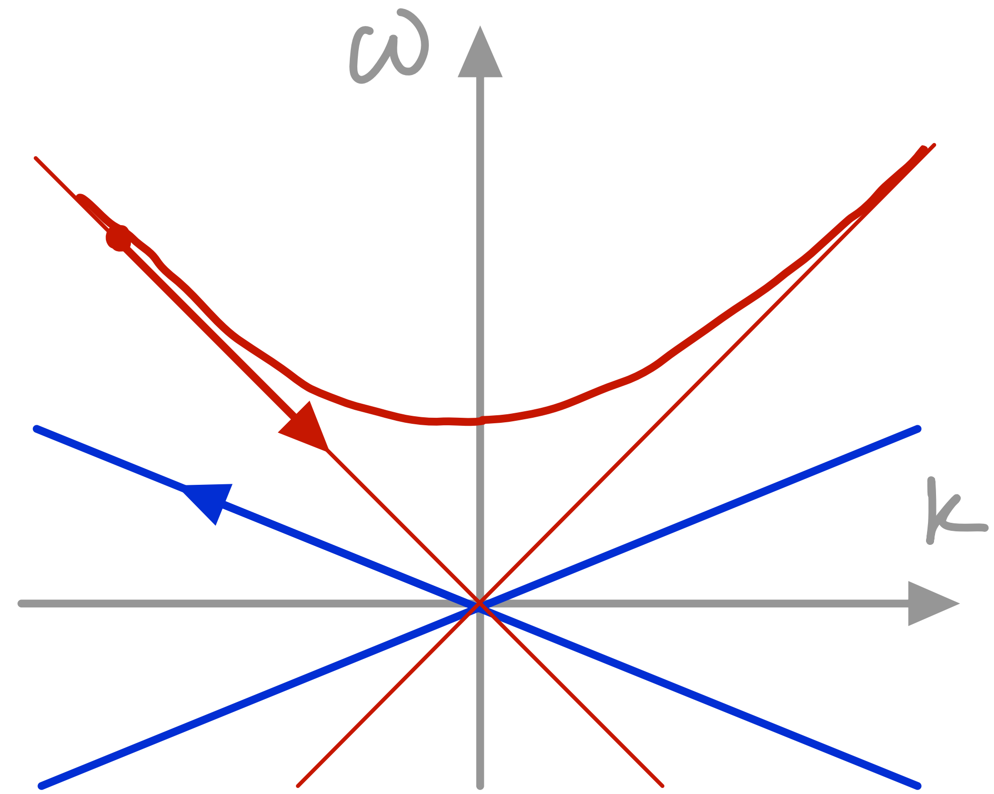
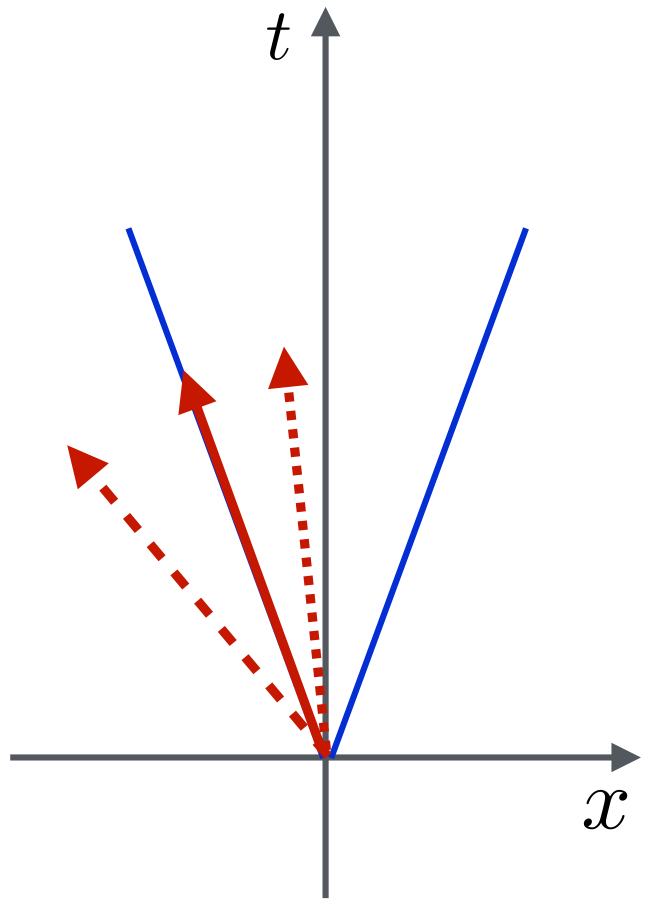
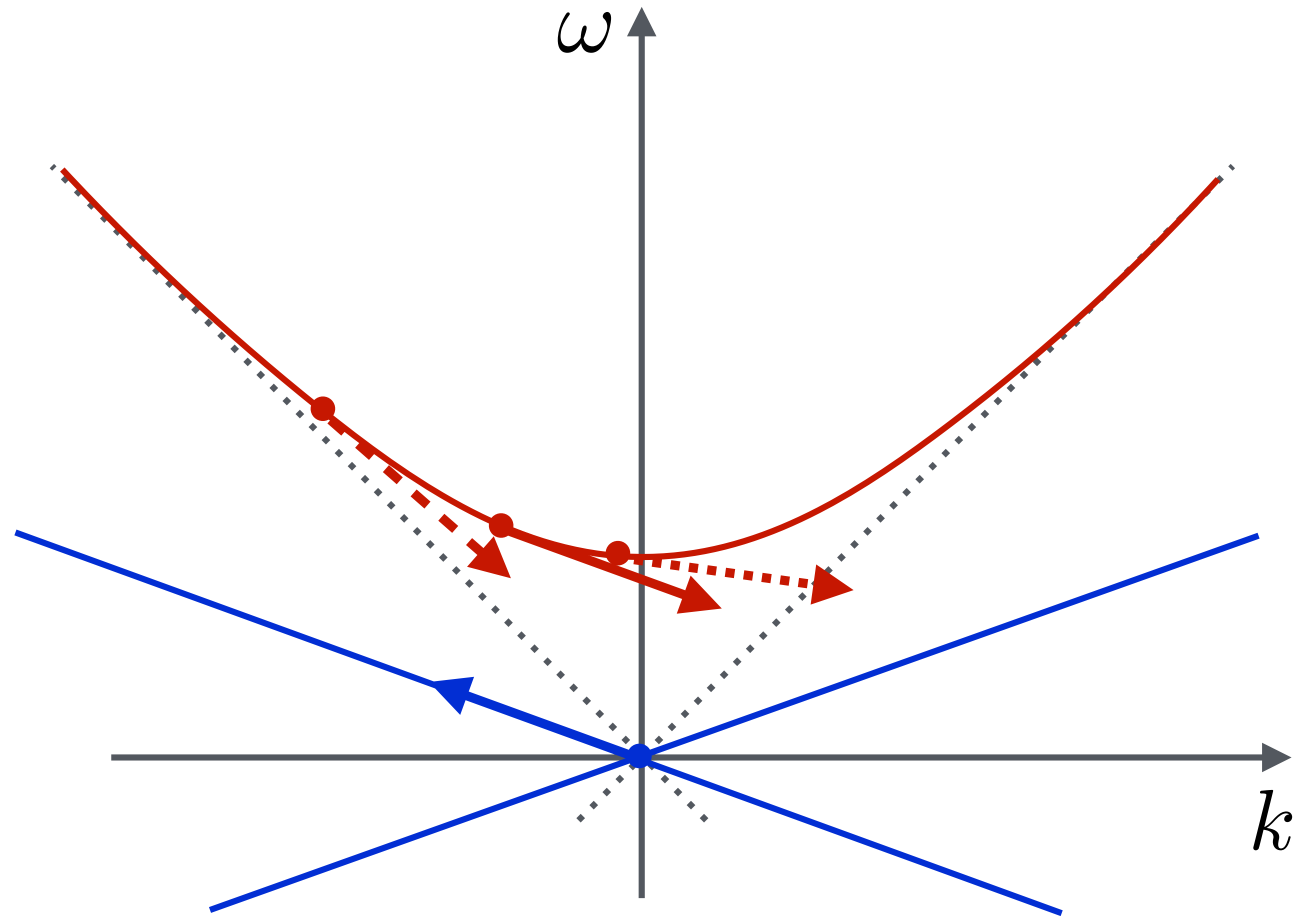




And now? No, in this case Cherenkov radiation is impossible in 1+1.

Cherenkov radiation





phonons: $k^\mu = \{c_s k, -k\}$

particle: $p^\mu = \left\{\sqrt{m^2+k^2}, -k\right\}$

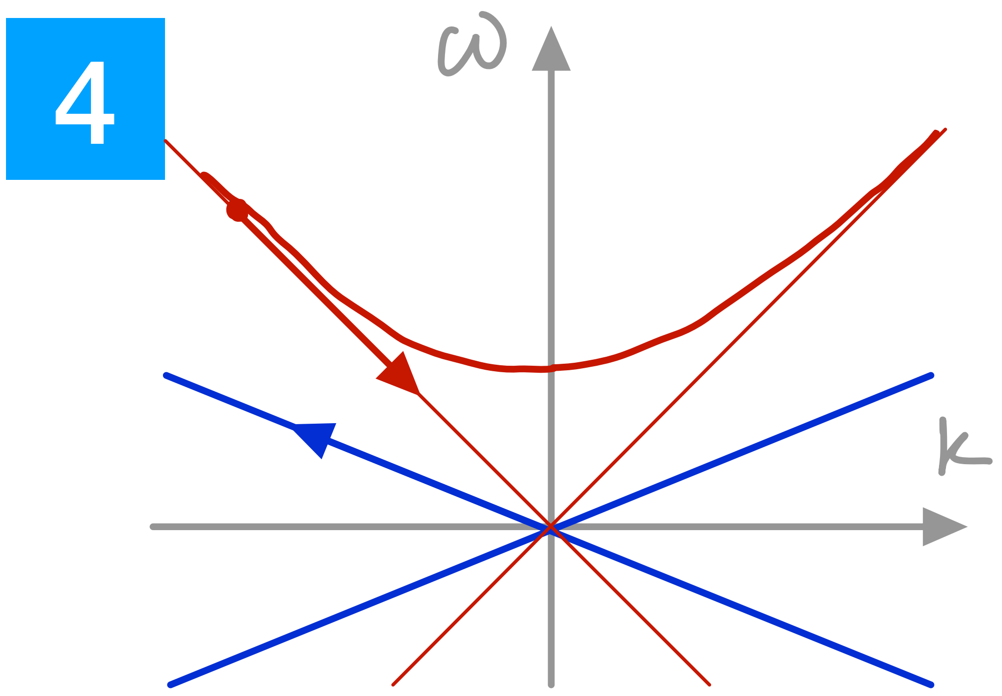
$p_1^\mu = p_2^\mu + k^\mu$

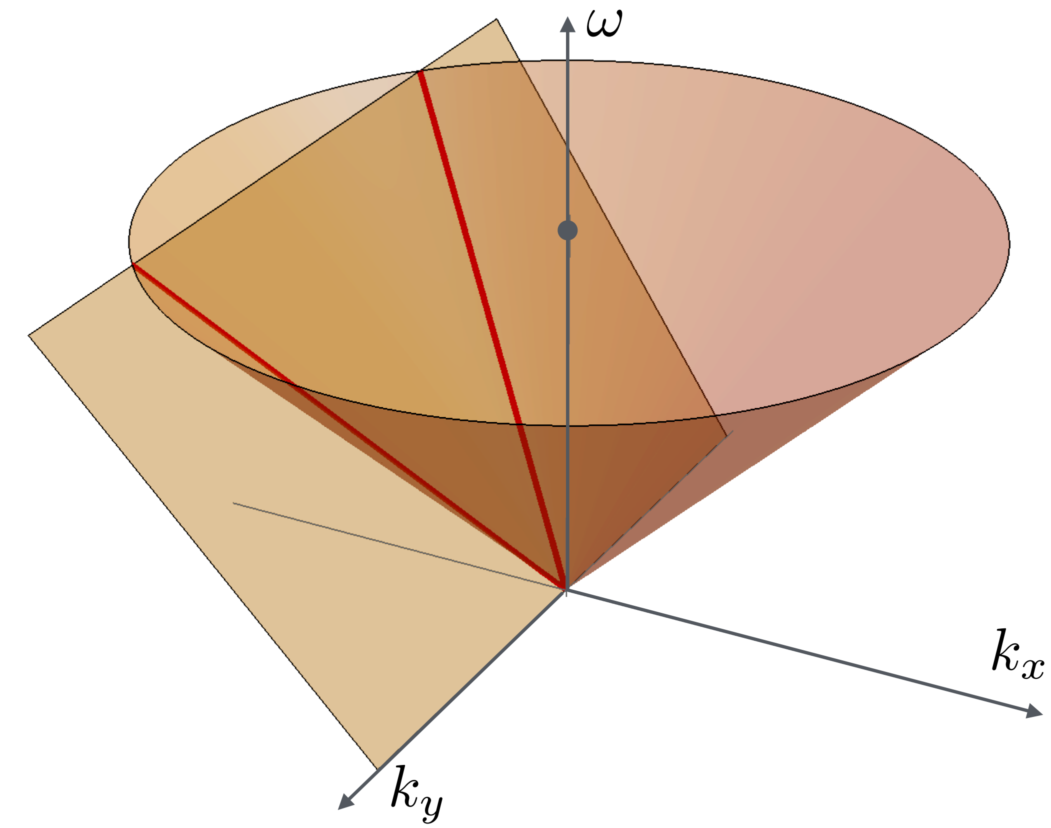
$0 = \Delta p^\mu + k^\mu$

where $\Delta p^\mu = p_2^\mu - p_1^\mu$

Cherenkov Radiation (1+1) (solid arrows)

Cherenkov radiation





Need (2+1). Heavy particle: tangential plane

to the mass shell at the point $\omega^2 = m^2 + k_x^2 + k_y^2$

Assume particles propagate in $x-$direction, the change in momentum is

$\Delta p^\mu = \{v\Delta p_x, \Delta p_x, \Delta p_y\}$

The emitted photons have $k^\mu = \left\{\pm c_s\sqrt{k_x^2 + k_y^2}, k_x, k_y\right\}$

Cherenkov radiation

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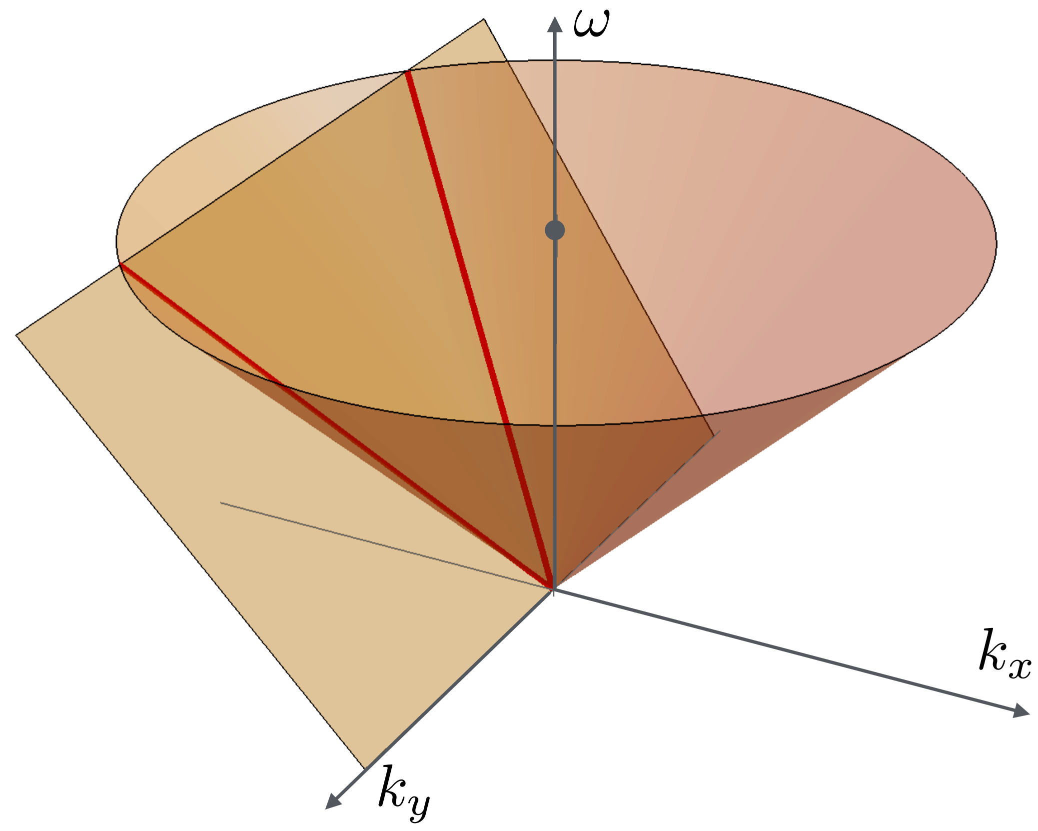


$$0 = \Delta p^\mu + k^\mu$$

where $\Delta p^\mu = p_2^\mu - p_1^\mu$

⇩

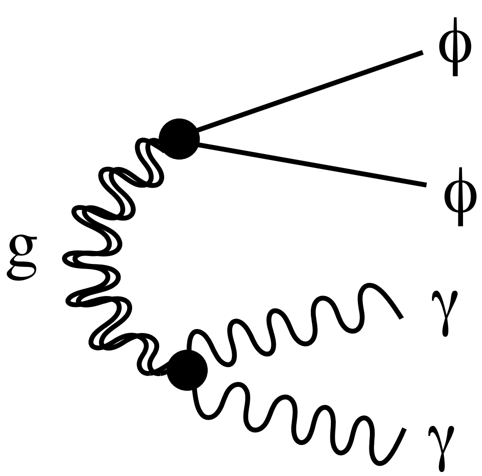
$$\Delta p^\mu = \{v\Delta p_x, \Delta p_x, \Delta p_y\}$$

$$k^\mu = \left\{\pm c_s\sqrt{k_x^2 + k_y^2}, k_x, k_y\right\}$$

$$\cos\theta = \frac{k_x}{\sqrt{k_x^2 + k_y^2}} = \frac{c_s}{v}$$

Cherenkov cone

Ghosts



Ghosts

Scalar field in Minkowski

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{m^2\phi^2}{2}$$

Canonical momentum $p = \frac{\partial\mathcal{L}_\phi}{\partial\dot{\phi}} = \dot{\phi}$

Hamiltonian

$$H = p\dot{\phi} - \mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\hat{\nabla}\phi) + \frac{m^2\phi^2}{2}$$

$H \geqslant 0$, bounded from below

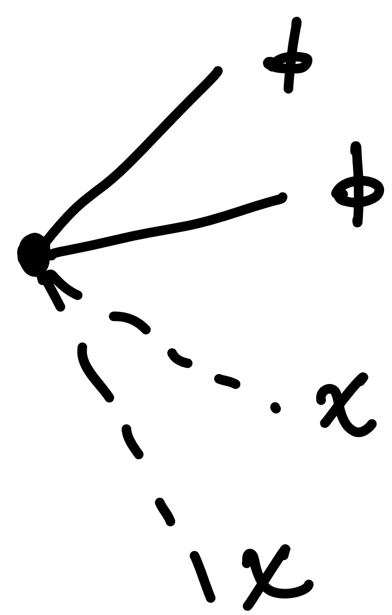
Ghosts

Consider a scalar with opposite sign

$$\mathcal{L}_{\text{ghost}} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi = -\frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\nabla\chi)^2$$

$$H_{\text{ghost}} = -\frac{1}{2}\dot{\chi}^2 - \frac{1}{2}(\bar{\nabla}\chi)^2 \leq 0$$ unbounded from below

- $\mathcal{L}_{\text{total}} = \mathcal{L}_\phi + \mathcal{L}_{\text{ghost}} + g\phi^2 x^2$



Vacuum decay

Standard problem of ghosts

Instability of black holes in scalar-tensor theories?

Perturbations of black holes in Horndeski theory (scalar-tensor theory of gravity) with time-dependent scalar field:

✦ Hamiltonian of perturbations (spherical symmetry):

$$H \sim \frac{1}{b_1}\left(\pi - \frac{1}{2}b_3\chi'\right)^2 + b_2\chi'^2$$

$b_1 > 0, \quad b_2 > 0.$ Boundedness from below

For interesting black hole solutions in the vicinity of the BH horizon, either b_1 or b_2 is negative $\Rightarrow$ **instability (?)**

Hamiltonian vs instability

Does unbounded from below Hamiltonian necessarily imply instability?

NO

GHOSTS by boosts

$$\mathcal{L} = \frac{1}{2}\dot{\chi}^2 - \frac{c_s^2}{2}\chi'^2$$

Relativistic boost $c = 1$:

$$\tilde{t} = \frac{t + vx}{\sqrt{1 - v^2}}, \quad \tilde{x} = \frac{x + vt}{\sqrt{1 - v^2}}$$

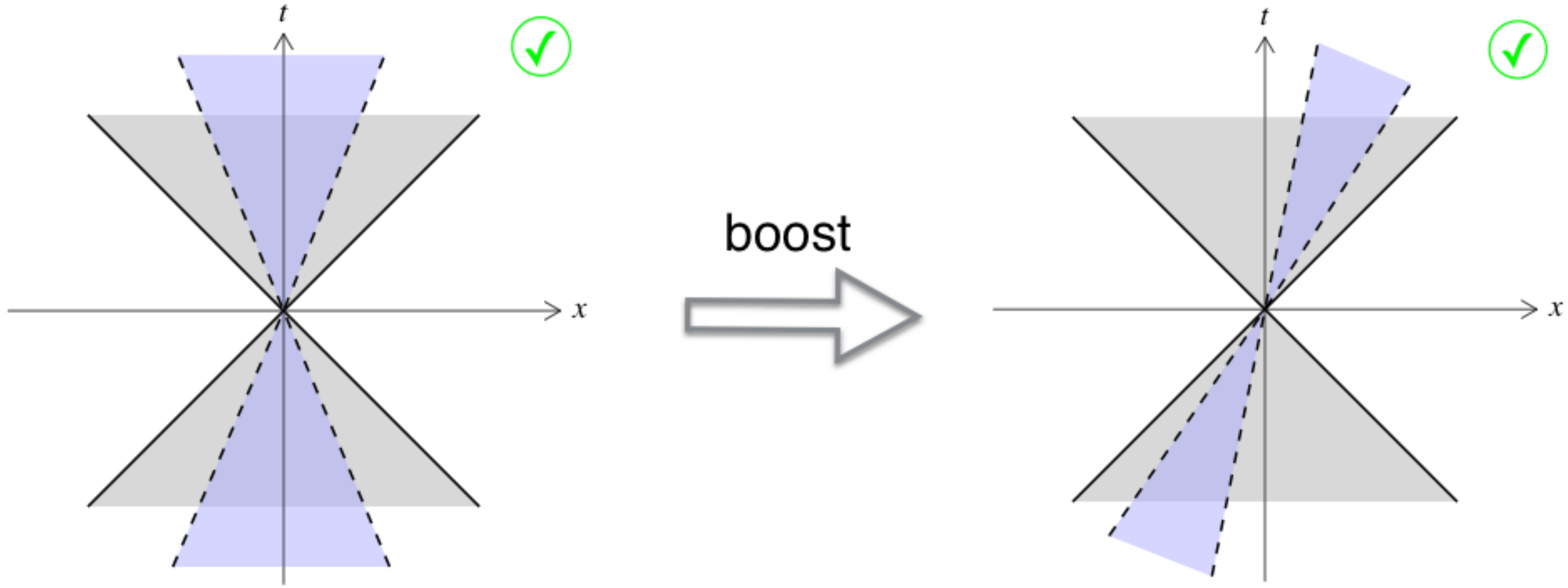
$$\mathcal{L} \to \frac{1}{1 - v^2}\left[\frac{1}{2}(1 - c_s^2 v^2)\dot{\chi}^2 + (1 - c_s^2)v\dot{\chi}\chi' - \frac{1}{2}(c_s^2 - v^2)\chi'^2\right]$$

Hamiltonian: $$\mathcal{H}_2 = \frac{1}{2}(\ldots)^2 + \frac{1}{2}(c_s^2 - v^2)\pi'^2$$

$\mathcal{H}_2 < 0$ for $|v| > c_s$

GHOSTS by boosts

non-GHOST



$$\mathcal{H}_2 = \frac{1}{2}(\dots)^2 + \frac{1}{2}c_s^2\pi'^2$$

$$\mathcal{H}_2 = \frac{1}{2}(\dots)^2 + \frac{1}{2}(c_s^2 - v^2)\pi'^2$$

need to compare 2 (or more) species

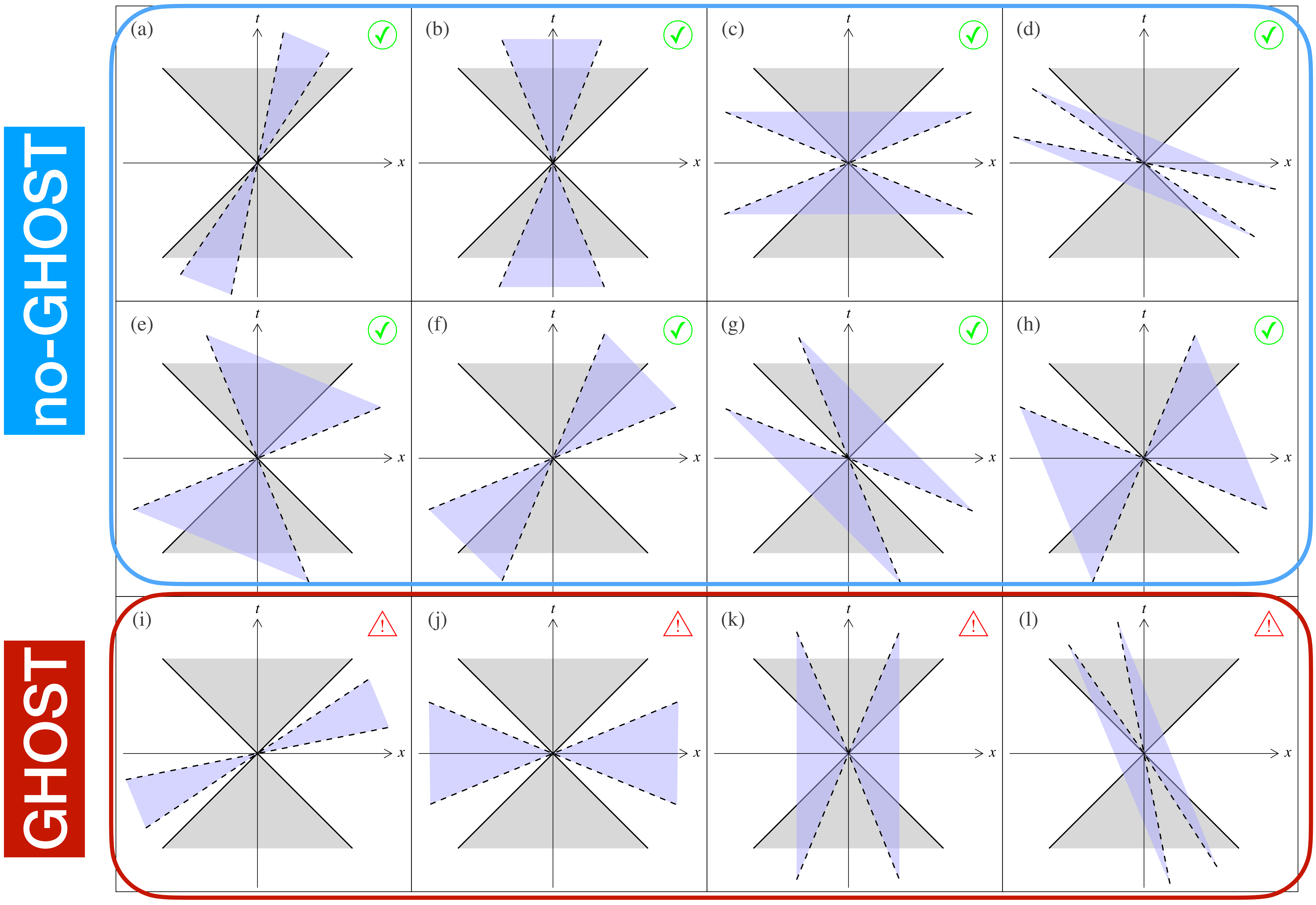
Stability vs Hamiltonian

- When total Hamiltonian density is bounded by below, then the lowest energy state is necessarily stable.
- Inverse is not true: A Hamiltonian density which is unbounded from below does not always imply instability (contrary to common lore).
- Sometimes the unbounded Hamiltonian appears due to the “bad” choice of coordinate

The Hamiltonian is not a scalar with respect to coordinate transformations

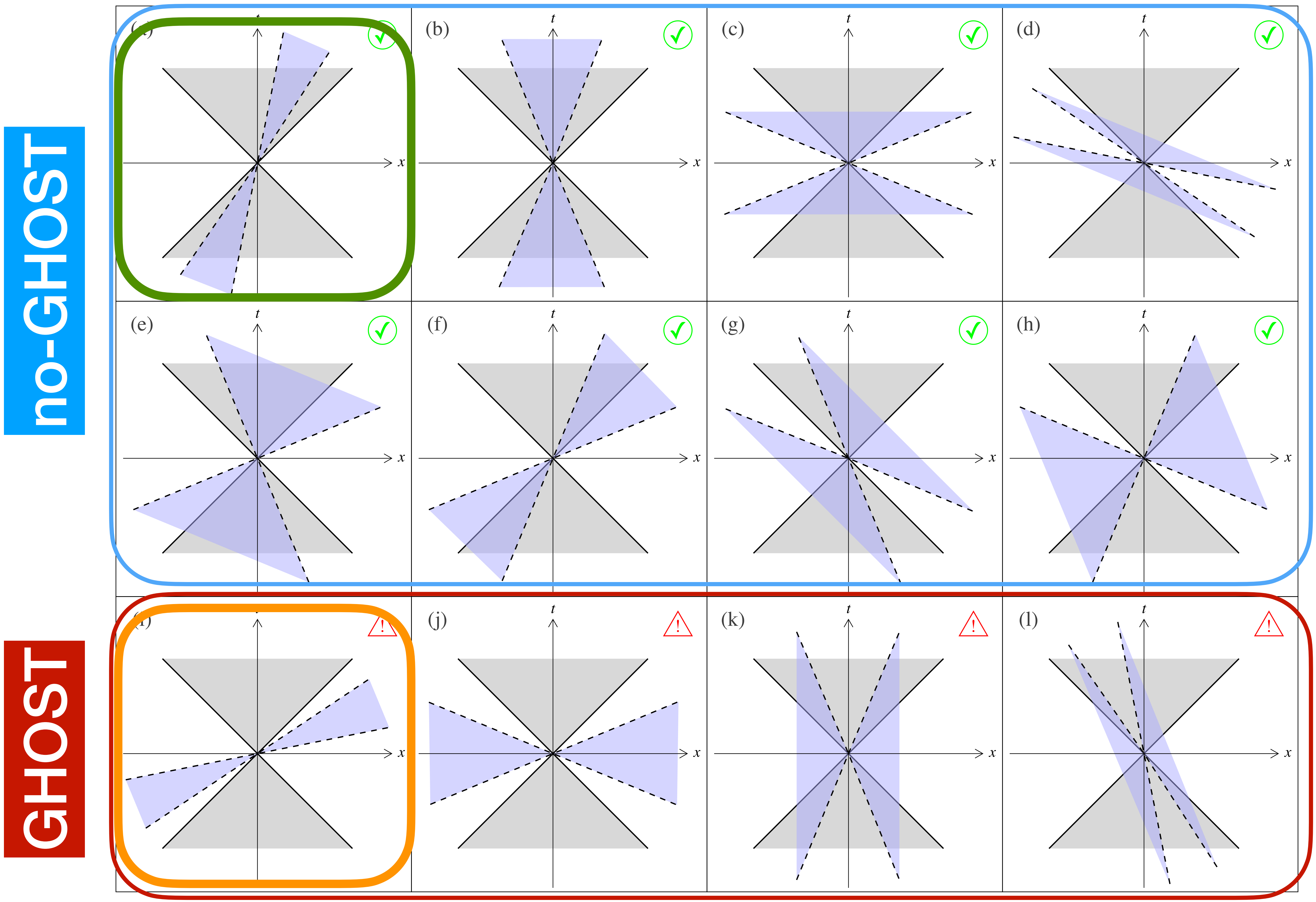
GHOSTS by boosts

[EB, Charmousis, Esposito-Farèse, Lehébel'17'18]

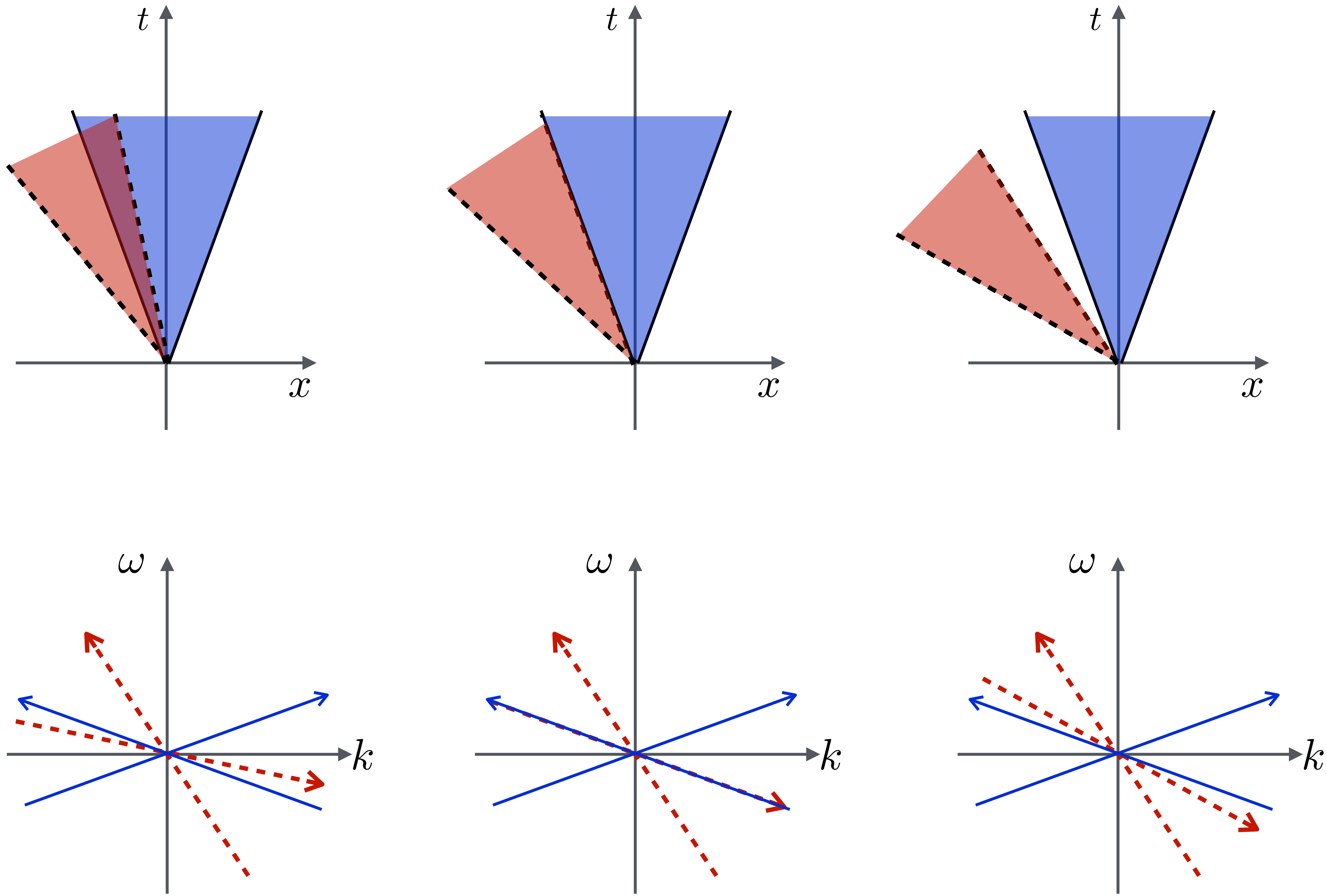


GHOSTS by boosts

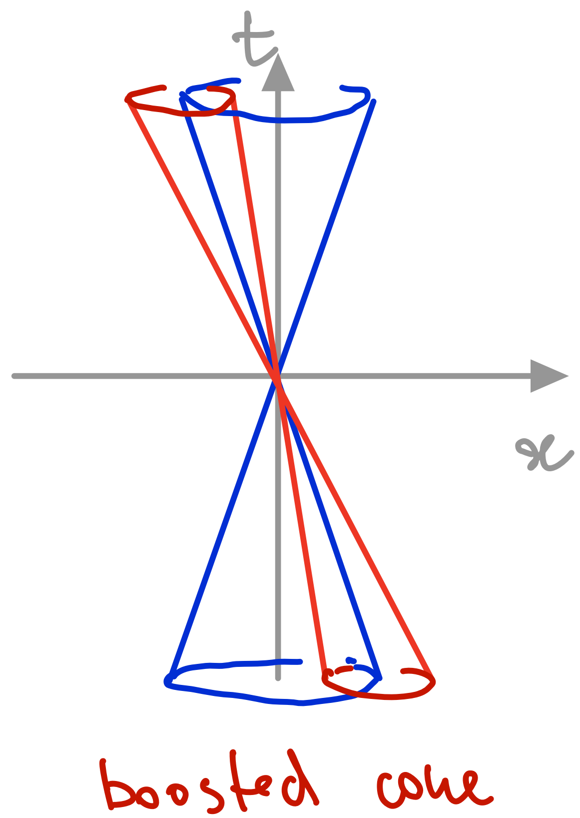
[EB, Charmousis, Esposito-Farèse, Lehébel’17’18]

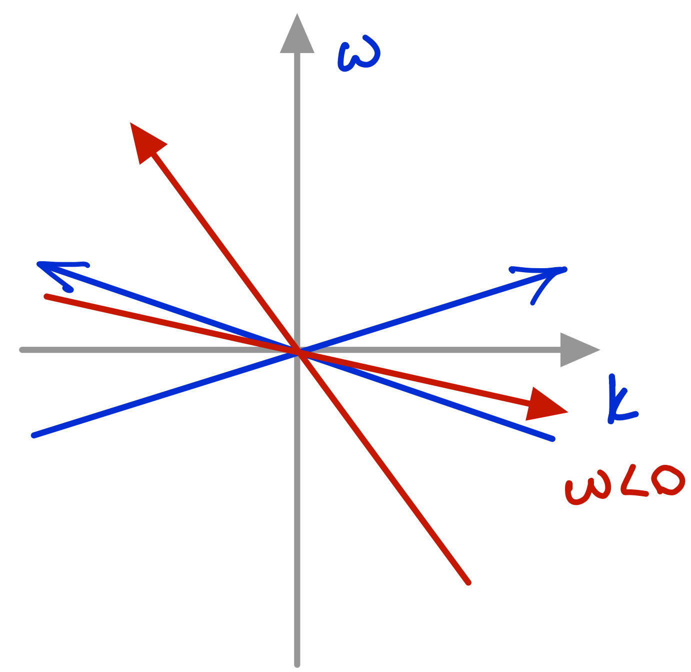


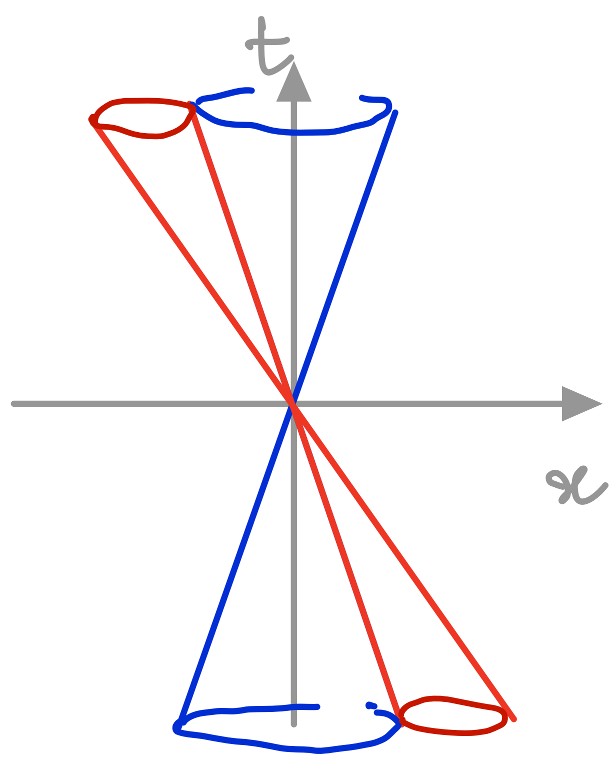
GHOSTS by boosts

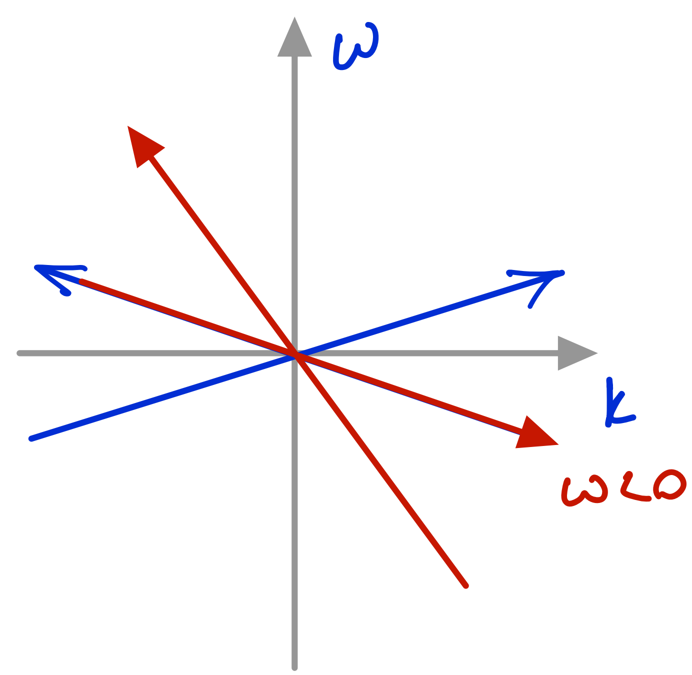


GHOSTS by boosts

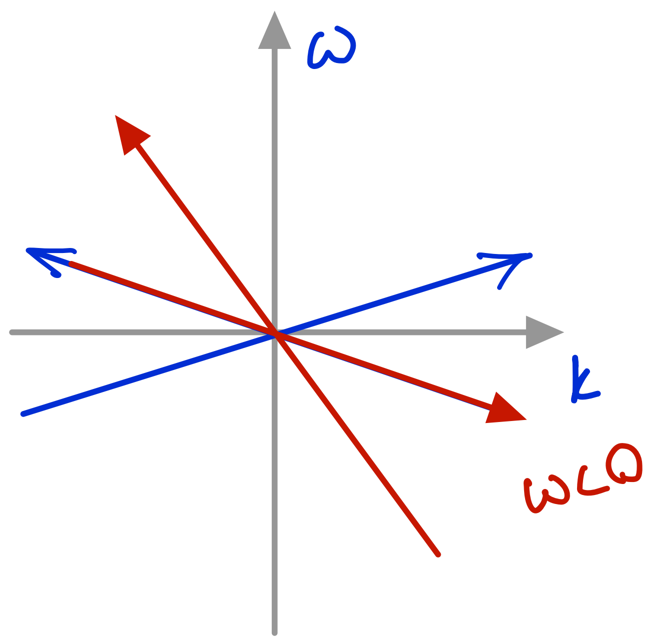






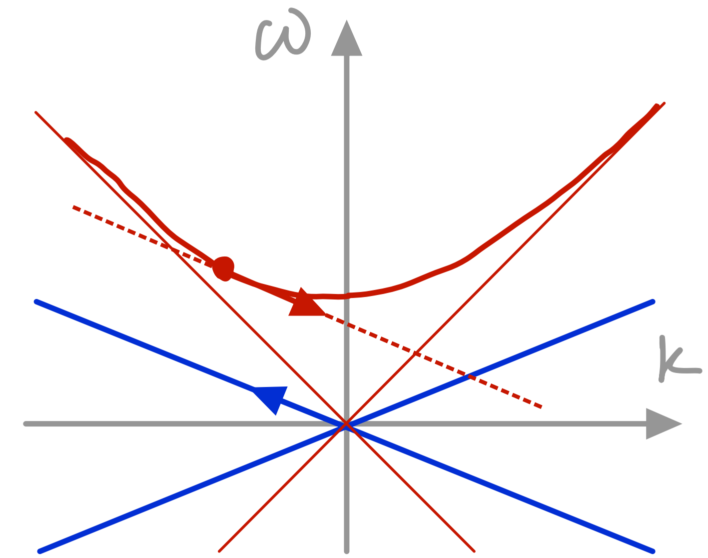


GHOST



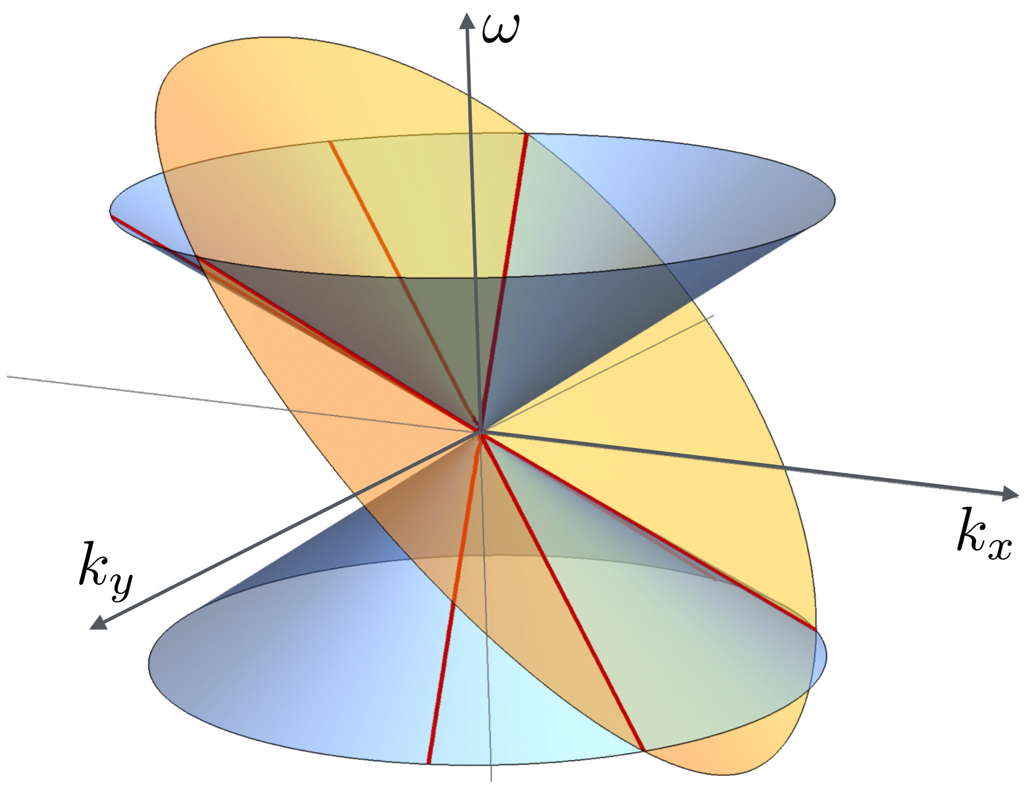
$$0 \to k^\mu + p^\mu$$

CHERENKOV

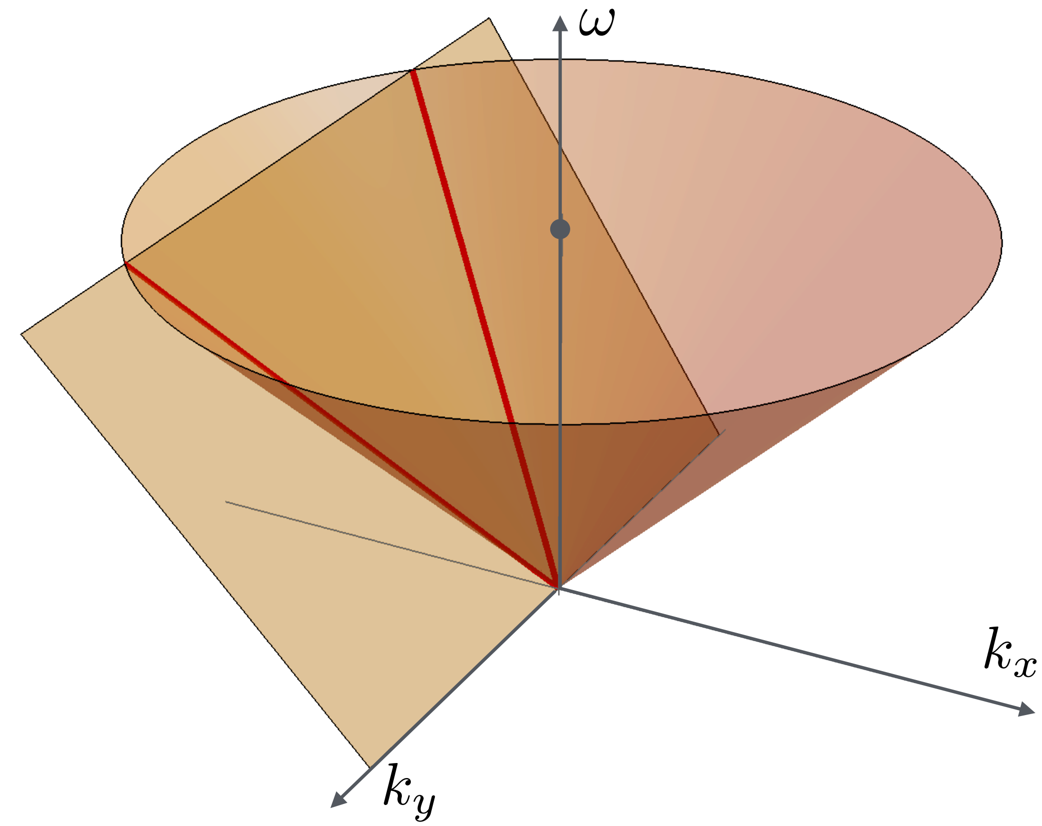


$$0 \to k^\mu + \Delta p^\mu$$

GHOST cone



Cherenkov cone



Cherenkov vs. Ghosts

Why is Cherenkov radiation ok (physical) ?

And ghosts are dangerous

- Cherenkov process is not instantaneous due to the physical cutoff
- Phonons cease to exist for very high energies, when the momentum is of order of the inverse distance between atoms
- In case of photons, the dispersion relation in medium is momentum-dependent, i.e. $c_s = c_s(k)$, so that the speed of photons grows as k increases
- There is no Cherenkov radiation for high energies of emitted particles

Cherenkov vs. Ghosts

Why is Cherenkov radiation ok (physical) ?

And ghosts are dangerous

- If a modified gravity theory is considered to be an EFT, then there is a cutoff of the theory, beyond which the description in terms of a specific modified gravity model becomes invalid
- The cutoff on the background solution determine the largest momenta of created pairs of normal particles and ghosts.
- Yet another limitation for the rate of a ghost instability: The background is eventually destroyed by the creation of particles on top of it

Conclusions

- Some ghost instabilities in modified gravity are fully analogous to Cherenkov radiation
- These ghosts are not "extremely" dangerous, although they do lead to instability of a background solution
- Ghost cones — can we observe them?